

BARNARD'S STAR - DISTANCE AND VELOCITY

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By combining measurements of parallax, proper motion (the apparent motion of a star across the sky, usually measured in seconds of arc per year) and the Doppler shifting of spectral lines, we can work out a star's actual velocity through space. A good example is Barnard's star, a magnitude 9.5 red dwarf star in the constellation Ophiucus, which has the largest known proper motion of any star.

The parallax is $p = 0.54901''$ so its distance is

$$d = \frac{1}{p} = 1.82146 \text{ pc} = 5.9408 \text{ ly} = 5.62032 \times 10^{16} \text{ m} \quad (1)$$

Its proper motion is $\mu = 10.3577'' \text{ yr}^{-1}$ which works out to

$$\mu = 10.3577 \frac{\pi}{3600 \times 180} = 5.021555 \times 10^{-5} \text{ rad yr}^{-1} \quad (2)$$

$$= \frac{5.021555 \times 10^{-5}}{3600 \times 24 \times 365.25} = 1.59123 \times 10^{-12} \text{ rad s}^{-1} \quad (3)$$

At the distance of Barnard's star, this gives a transverse velocity component of

$$v_{\theta} = d\mu = 8.9432 \times 10^4 \text{ m s}^{-1} \quad (4)$$

In the spectrum, the hydrogen alpha ($\text{H}\alpha$) line is observed at a wavelength of 656.034 nm, and the rest $\text{H}\alpha$ wavelength is 656.28 nm. As the star's wavelength is shorter than the rest wavelength, it is blue-shifted and Barnard's star is moving towards us. The (relativistic) Doppler shift formula for a speed v that is positive for approaching objects is

$$\bar{\lambda} = \lambda \sqrt{\frac{1 - v/c}{1 + v/c}} \quad (5)$$

For non-relativistic speeds, $v \ll c$ and we can Taylor-expand this formula to get

$$\bar{\lambda} = \lambda - \frac{\lambda}{c}v + \mathcal{O}\left(\frac{v^2}{c^2}\right) \quad (6)$$

Keeping only terms first order in v/c we get

$$v = \frac{\lambda - \bar{\lambda}}{\lambda}c \quad (7)$$

For Barnard's star this gives a radial velocity of

$$v_r = \frac{656.28 - 656.034}{656.28} (2.99792 \times 10^8) \quad (8)$$

$$= 1.12374 \times 10^5 \text{ m s}^{-1} \quad (9)$$

The actual speed of Barnard's star through space, relative to Earth, is

$$v = \sqrt{v_r^2 + v_\theta^2} = 1.4362 \times 10^5 \text{ m s}^{-1} \quad (10)$$