

COLOUR INDICES IN TERMS OF BLACKBODY RADIATION RATE

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog and include the title or URL of this post in your comment.

Post date: 9 Jun 2023.

We can write the apparent magnitude m_λ of a star for some wavelength band in terms of the flux F_λ in that band received on Earth from the star as

$$m_\lambda = -2.5 \log \int F_\lambda S_\lambda d\lambda + C_\lambda \quad (1)$$

The constant C_λ depends on the wavelength interval over which the integral is done, and on the sensitivity S_λ of the detector, so it will be different for different filters. If we measure the apparent magnitudes in the three standard bands U , B and V and treat the star as a blackbody, we can work out the three constants C_U , C_B and C_V . Since colour indices are commonly used to classify stars, we can work out similar equations for these indices.

$$U - B = -2.5 \log \int F_U S_U d\lambda + C_U + 2.5 \log \int F_B S_B d\lambda - C_B \quad (2)$$

$$= -2.5 \log \frac{\int F_U S_U d\lambda}{\int F_B S_B d\lambda} + C_{U-B} \quad (3)$$

where $C_{U-B} \equiv C_U - C_B$. Since the argument of the log is now dimensionless, the constant C_{U-B} is independent of the units used to measure flux. In their example 3.6.2, Carroll & Ostlie use a star with surface temperature of $T = 42000$ K and measured colour indices of $U - B = -1.19$ and $B - V = -0.33$. The standard filters are

- U : 365 ± 34 nm;
- B : 440 ± 49 nm;
- V : 550 ± 44.5 nm.

If we make the assumptions that $S = 1$ for each filter within these bands and $S = 0$ outside these bands, and that the flux doesn't change much over the bandwidth in each filter, we can approximate the relation above by

$$U - B \approx -2.5 \log \frac{F_U \Delta\lambda_U}{F_B \Delta\lambda_B} + C_{U-B} \quad (4)$$

with a similar relation for $B - V$:

$$B - V \approx -2.5 \log \frac{F_B \Delta \lambda_B}{F_V \Delta \lambda_V} + C_{B-V} \quad (5)$$

As we're dealing with ratios of flux for the same star, we can express these equations in terms of the blackbody radiation rate:

$$U - B = -2.5 \log \frac{B_U \Delta \lambda_U}{B_B \Delta \lambda_B} + C_{U-B} \quad (6)$$

$$= -2.5 \log \frac{\lambda_B^5 \left(e^{hc/\lambda_B k_B T} - 1 \right) \Delta \lambda_U}{\lambda_U^5 \left(e^{hc/\lambda_U k_B T} - 1 \right) \Delta \lambda_B} + C_{U-B} \quad (7)$$

$$B - V = -2.5 \log \frac{\lambda_V^5 \left(e^{hc/\lambda_V k_B T} - 1 \right) \Delta \lambda_B}{\lambda_B^5 \left(e^{hc/\lambda_B k_B T} - 1 \right) \Delta \lambda_V} + C_{B-V} \quad (8)$$

Plugging in the values for the star mentioned above, we get

$$C_{U-B} = -0.87 \quad (9)$$

$$C_{B-V} = +0.65 \quad (10)$$

Given these values, we can now estimate the colour indices for any star if we know its temperature. For the Sun, $T = 5777$ K and plugging this into 7 and 8, we get

$$U - B = -0.222 \quad (11)$$

$$B - V = +0.571 \quad (12)$$

The measured values given by Carroll & Ostlie are $U - B = +0.195$ and $B - V = +0.65$ so the agreement isn't great, especially for $U - B$.