

COMPTON WAVELENGTH

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The Compton effect is the change in the frequency of a photon when it scatters off a stationary electron. The formula, in relativistic units ($c = 1$) is

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m}(1 - \cos\theta) \quad (1)$$

To convert this formula to wavelengths and to restore c , we first reinsert c into the above equation. The units of ν are s^{-1} , and of m are kg^{-1} . The units of Planck's constant can be found from its definition in the formula

$$E = h\nu \quad (2)$$

Thus the units of h are $(\text{energy}) \times (\text{time}) = \text{kg m}^2\text{s}^{-1}$. This also happens to be the units of angular momentum, since $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, so the units of \mathbf{L} are $\text{m} \times \text{kg m s}^{-1} = \text{kg m}^2\text{s}^{-1}$. From 1, the units of the last term on the RHS must be seconds, so with c present explicitly, we must have

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{mc^2}(1 - \cos\theta) \quad (3)$$

In terms of wavelength, we have

$$\lambda\nu = c \quad (4)$$

so

$$\frac{\lambda'}{c} = \frac{\lambda}{c} + \frac{h}{mc^2}(1 - \cos\theta) \quad (5)$$

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta) \quad (6)$$

The change in wavelength is inversely proportional to the mass of the stationary object off which the photon scatters. For an electron

$$\frac{h}{m_e c} = \frac{6.62606957 \times 10^{-34}}{(9.10938291 \times 10^{-31})(2.99792458 \times 10^8)} = 2.426 \times 10^{-12} \text{ m} \quad (7)$$

This is the *Compton wavelength* of the electron. The wavelength shift is independent of the wavelength of the incoming photon, so for visible light, where λ is around 5×10^{-7} m, the Compton effect is negligible. It becomes noticeable only for much shorter wavelengths (and thus much higher energy photons). For X-rays and gamma rays, λ is in the range $10^{-12} - 10^{-11}$ m so the Compton shift is comparable with the original wavelength.

For a heavier particle, such as a proton, the Compton effect is correspondingly smaller. The Compton wavelength of a proton is

$$\frac{h}{m_p c} = \frac{6.62606957 \times 10^{-34}}{(1.67262178 \times 10^{-27})(2.99792458 \times 10^8)} = 1.321 \times 10^{-15} \text{ m} \quad (8)$$