

GRAVITATIONAL HYDROGEN ATOM

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To see that gravity is much weaker than the electrostatic force, we can repeat Bohr's semi-classical derivation of the hydrogen energy levels, replacing the Coulomb force with the Newtonian gravitational force. We can do this with the following replacement:

$$\frac{e^2}{4\pi\epsilon_0} \rightarrow Gm_em_p \quad (1)$$

giving energy levels of

$$E = -(Gm_em_p)^2 \frac{m_e}{2n^2\hbar^2} \quad (2)$$

$$= -\frac{4.23 \times 10^{-97} \text{ J}}{n^2} \quad (3)$$

$$= -\frac{2.64 \times 10^{-78} \text{ eV}}{n^2} \quad (4)$$

compared to the actual energy levels of hydrogen:

$$E = -\frac{13.6 \text{ eV}}{n^2} \quad (5)$$

The radii are

$$r_n = \frac{n^2\hbar^2}{Gm_e^2m_p} \quad (6)$$

$$= (1.2 \times 10^{29} \text{ m}) n^2 \quad (7)$$

$$= (1.2 \times 10^{38} \text{ nm}) n^2 \quad (8)$$

$$= (8 \times 10^{17} \text{ AU}) n^2 \quad (9)$$

$$= (1.27 \times 10^{13} \text{ ly}) n^2 \quad (10)$$

Thus the ground state radius of gravitational hydrogen is many times larger than the visible universe, compared with the electrostatic Bohr radius of $5.29177 \times 10^{-11} \text{ m}$.