OPACITY AND OPTICAL DEPTH

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We've looked at the mean free path of gas molecules and it's reasonable to ask if we can apply the same reasoning to the propagation of light through a gas. The difficulty with this is that a photon can not only be scattered by gas molecules, it can also be absorbed, so that a single photon might not even survive its journey through the gas.

Opacity

To deal with this, a more general treatment of the passage of light through a gas is used. We consider a ray of light of a given intensity I_λ, where the subscript λ indicates that we're considering a single wavelength. The intensity of the ray will in general lose an amount dI_λ of its value as it passes through a thickness ds of gas. If we consider the fraction dI_λ/I_λ of intensity that is lost, this will depend on the density ρ of gas and on the absorptive and/or scattering properties of the gas. The latter quantity is called the *opacity* and is given the symbol κ_λ, again depending on the wavelength. Thus the general relationship is

$$\frac{dI_\lambda}{I_\lambda}=-\kappa_\lambda\rho\,ds \tag{1}$$

where the minus sign indicates that the intensity decreases with increasing distance through the gas. The general solution is therefore

$$I_\lambda(s)=I_\lambda(0)\,e^{-\int\kappa_\lambda\rho\,ds} \tag{2}$$

In general, both κ_λ and ρ can vary along the path of the light ray. In the special case where they are both constant, we have

$$I_\lambda(s)=I_\lambda(0)\,e^{-\kappa_\lambda\rho s} \tag{3}$$

The quantity $\kappa_\lambda\rho$ has the dimensions of inverse length, so that $\ell\equiv1/\kappa_\lambda\rho$ is the distance over which the intensity decreases by a factor of e. We can take this to be a measure of how far into the gas we can see.

Example. For the Sun at visible wavelengths (say, $\lambda = 500$ nm) the value of $\kappa_{500} = 0.03\ \mathrm{m^2 kg^{-1}}$ and the Sun's photosphere (outer atmosphere) has a density of $\rho = 2.1 \times 10^{-4}\ \mathrm{kg\ m^{-3}}$, so the length ℓ is

$$\ell_{\odot} = \frac{1}{\kappa_{500}\rho} = 1.587 \times 10^5\ \mathrm{m} = 158.7\ \mathrm{km} \tag{4}$$

If the Earth's atmosphere (density of about $1.2\ \mathrm{kg\ m^{-3}}$) had the same opacity at visible wavelengths, the length would be

$$\ell_{\oplus} = 27.7\ \mathrm{m} \tag{5}$$

Thus the opacity of the Earth's atmosphere at visible wavelengths must be considerably less than that of the Sun, as we can usually see several kilometres on a clear day.

Optical depth

Another measure of the optical property of a gas is given by the *optical depth* τ_λ. This is a measure of the loss of intensity over some finite distance. It's defined from 1 as

$$d\tau_\lambda = -\kappa_\lambda \rho\, ds \tag{6}$$

so that the fractional loss of intensity is given by

$$\frac{dI_\lambda}{I_\lambda} = d\tau_\lambda \tag{7}$$

From its definition, we see that τ_λ is dimensionless (since $\kappa_\lambda \rho$ has dimensions of inverse length and ds is a length increment). The optical depth of a region between $s = 0$ and some other value of s is therefore

$$\Delta\tau_\lambda = \tau_\lambda(s) - \tau_\lambda(0) = -\int_0^s \kappa_\lambda \rho\, ds \tag{8}$$

Note that $\Delta\tau_\lambda$ is negative when applied to a star, since the $s = 0$ point is usually located somewhere inside the star and the other value of s is often taken as outside the star. If the light ray travels through a vacuum after leaving the star, it's convenient to take $\tau_\lambda(s) = 0$ and to refer to $\tau_\lambda(0)$ as just τ_λ, which is the optical depth of a point within the star relative to empty space. In this case

$$\tau_\lambda = \int_0^s \kappa_\lambda \rho\, ds \tag{9}$$

Note that in this case $\tau_{\lambda}>0$, since it represents the optical depth of a point inside the star as viewed from outside. In the special case where the opacity and density are both constant, we have

$$\tau_{\lambda}=\kappa_{\lambda}\rho s \tag{10}$$

where s is the depth within the star.

The intensity can then be written as

$$I_{\lambda}(s)=I_{\lambda}(0)\,e^{-\tau_{\lambda}} \tag{11}$$

At an optical depth of $\tau_{\lambda}=1$, the intensity has dropped to a fraction e^{-1} of its initial value. We can therefore interpret the optical depth as roughly how far we can see into an absorbing medium. Looked at another way, τ_{λ} is roughly the number of mean free path lengths in a straight line along the photon's path.

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