OPTICAL DEPTH OF EARTH'S ATMOSPHERE

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The optical depth is a measure of how far we see through a medium that absorbs and/or scatters light. The optical depth $d\tau_\lambda$ of a thickness ds of a medium with opacity κ_λ and density ρ is defined as

$$\tau_\lambda = \int_0^s \kappa_\lambda \rho \, ds \tag{1}$$

The subscript λ indicates that opacity, and hence optical depth, can depend on the wavelength of the light. The fractional loss of intensity due to opacity is written in terms of the optical depth as

$$\frac{dI_\lambda}{I_\lambda} = -d\tau_\lambda \tag{2}$$

(the minus sign is because the intensity drops with increasing optical depth) so we have

$$I_\lambda(s) = I_\lambda(0)\, e^{-\tau_\lambda(s)} \tag{3}$$

That is, the intensity of a light ray after passing through a distance s of a medium with optical depth $\tau_\lambda(s)$ decreases exponentially.

We can apply this concept to the intensity of light from a star as it passes through Earth's atmosphere. We take $s = 0$ at the ground (the location of the telescope), and $I_{\lambda,*}$ to be the intensity of the starlight before it encounters the atmosphere. If we're observing the star from a ground-based telescope, the distance travelled by the starlight depends on the angle θ that the observation line of sight makes with the vertical. If a star is directly overhead, its light travels the minimum distance (call it s_0) through the atmosphere, so we can call the optical depth at that angle $\tau_{\lambda,0}$. If we observe the star at an angle θ, the distance travelled is $s_0/\cos\theta = s_0 \sec\theta$. In that case, from 1, the vertical distance $dz = \cos\theta \, ds$, and we have

$$\tau_{\lambda,\theta} = \int_0^s \kappa_\lambda \rho \, ds = \int_0^s \kappa_\lambda \rho \frac{dz}{\cos\theta} = \sec\theta \int_0^s \kappa_\lambda \rho \, dz = \tau_{\lambda,0} \sec\theta \tag{4}$$

From 3 we therefore have

$$I_{\lambda}(s) = I_{\lambda}(0)\, e^{-\tau_{\lambda,0}\sec\theta} \quad (5)$$

In practice, only $I_{\lambda}(s)$ and θ are known from a single observation. However, if we observe at two different angles (say at two different times, so that the rotation of the Earth causes the angle of observation to change) θ_1 and θ_2, we can take logs of 5 to get

$$\ln I_{\lambda}(s_i) = \ln I_{\lambda}(0) - \tau_{\lambda,0}\sec\theta_i \quad (6)$$

where $i = 1,2$. Plotting $I_{\lambda}(s)$ versus $\sec\theta$ gives a straight line with slope $-\tau_{\lambda,0}$ and intercept $\ln I_{\lambda}(0)$. Thus from two measurements, we have

$$\tau_{\lambda,0} = -\frac{\ln I_{\lambda}(s_2) - \ln I_{\lambda}(s_1)}{\sec\theta_2 - \sec\theta_1} \quad (7)$$

$$I_{\lambda}(0) = I_{\lambda}(s_i)\, e^{\tau_{\lambda,0}\sec\theta_i} \quad (8)$$

where we use the value of $\tau_{\lambda,0}$ from the first equation to calculate $I_{\lambda}(0)$ in the second. In the second equation we can take i to be either 1 or 2. Note that 7 gives a positive value for $\tau_{\lambda,0}$, since if $\theta_2 > \theta_1$, the thickness $s_2 > s_1$, and therefore $I_{\lambda}(s_2) < I_{\lambda}(s_1)$, since the light travels through a greater thickness.

Note that because $\sec\theta \geq 1$ for all angles, the intercept on the graph must be obtained by extrapolating the line to the vertical axis, even though no actual measurement can be done at that point.