PATH OF A PHOTON IN THE SUN

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We can get an estimate for how far a photon travels, and how long it takes, for it to reach the surface of the Sun starting from the centre. For this, we'll use the Rosseland mean opacity $\overline{\kappa}$ and the density ρ of the Sun at its centre. Carroll & Ostlie give values for these in Problem 9.11:

$$\begin{aligned}\overline{\kappa} &= 0.217 \text{ m}^2\text{kg}^{-1} \\ \rho &= 1.53\times 10^5\text{kg m}^{-3}\end{aligned} \tag{1}$$

The reasoning uses the theory of random walks. Due to repeated absorption and emission, and to repeated scattering, a photon executes a random walk as it traverses the interior of the Sun. We can approximate the process by assuming that each step in the walk has the same length ℓ, the mean free path of the photon. We can then define a vector $\boldsymbol{\ell}_i$ for the ith step in the walk, so that the net distance travelled is

$$\mathbf{d} = \sum_{i=1}^{N} \boldsymbol{\ell}_i \tag{2}$$

where N is the number of steps taken.

The actual distance travelled can then be written as

$$d^2 = \mathbf{d}\cdot\mathbf{d} \tag{3}$$

$$= \sum_{i,j=1}^{N} \boldsymbol{\ell}_i\cdot\boldsymbol{\ell}_j \tag{4}$$

This sum will contain N terms, each of magnitude ℓ^2, and N^2-N terms containing a cosine of the angle between two $\boldsymbol{\ell}_i$ vectors. For very large N, we'd expect the sum of these cosines to be approximately zero, since the vectors are all in random directions. Thus we have

$$d^2 \approx N\ell^2 \tag{5}$$

We now consider the optical depth τ. Using the optical depth as a measure of the number of mean free paths along a straight line from the centre of the Sun to the surface, we have

$$d = \tau \ell \tag{6}$$

Combining this with 5 we have

$$\tau = \sqrt{N} \tag{7}$$

If we assume that the values 1 are constant throughout the Sun, and that we can use the Rosseland mean opacity for κ (certainly not true in general, but we're just getting an estimate), then from the definition of τ, we have

$$\tau = \overline{\kappa}\rho d \tag{8}$$

The radius of the Sun is

$$d = 6.9634 \times 10^8 \text{ m} \tag{9}$$

We get

$$\tau = 2.31 \times 10^{13} \tag{10}$$

The number of mean free paths in the random walk from the centre to the surface is then

$$N = \tau^2 = \left(2.31 \times 10^{13}\right)^2 = 5.34 \times 10^{26} \tag{11}$$

A single mean free path can be found from 6.

$$\ell = \frac{d}{\tau} = 3 \times 10^{-5} \text{ m} \tag{12}$$

We can then get an estimate for how long it takes a photon to get from the centre to the surface of the Sun.

$$t = \frac{N\ell}{c} = 5.37 \times 10^{13} \text{ s} = 1.7 \times 10^6 \text{ years} \tag{13}$$

This value is about a factor of 10 larger than the currently accepted value of about 170,000 years, but we assumed that both the optical density and mass density of the Sun are constant all the way to the surface, which is certainly not true. We'd expect both densities to decrease as we neared the surface, which would lengthen the mean free path and thus reduce N (in proportion to τ^2) and thus the time required to escape.