PHOTON DENSITY IN A BLACKBODY

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The energy density for a blackbody is given in Carroll & Ostlie Chapter 9 as

$$u_\lambda d\lambda = \frac{4\pi}{c} B_\lambda d\lambda \tag{1}$$

$$= \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT}-1} d\lambda \tag{2}$$

This is the energy per m^3 for wavelengths between λ and $\lambda + d\lambda$.

The energy of a single photon with frequency ν is given by Planck's formula

$$E_\nu = h\nu \tag{3}$$

and the frequency is related to the wavelength by

$$\lambda\nu = c \tag{4}$$

so from 2, the number density n_λ of photons in a blackbody is

$$n_\lambda d\lambda = \frac{u_\lambda}{h\nu} d\lambda \tag{5}$$

$$= \frac{\lambda u_\lambda}{hc} d\lambda \tag{6}$$

$$= \frac{8\pi/\lambda^4}{e^{hc/\lambda kT}-1} d\lambda \tag{7}$$

where

$$\begin{aligned} h &= 6.626 \times 10^{-34} \text{ J s} \\ k &= 1.38 \times 10^{-23} \text{ J K}^{-1} \\ c &= 3 \times 10^{8} \text{ m s}^{-1} \end{aligned} \tag{8}$$

h is Planck's constant, k is Boltzmann's constant and c is the speed of light, all in SI units. The units of $n_\lambda d\lambda$ are m^{-3}, that is, number of photons per cubic metre.

The total photon density is obtained by integrating 7, which we can do with Maple.

$$n = \int_0^\infty n_\lambda d\lambda \tag{9}$$

$$= \int_0^\infty \frac{8\pi/\lambda^4}{e^{hc/\lambda kT} - 1} d\lambda \tag{10}$$

$$= 16\pi \left(\frac{kT}{hc}\right)^3 \zeta(3) \tag{11}$$

where ζ is the Riemann zeta function, with the value

$$\zeta(3.0) = 1.202056903 \tag{12}$$

Plugging in the physical constants we get

$$n = 2.0245 \times 10^7 T^3 \tag{13}$$

Example 1. The total number of photons in a kitchen oven at a temperature of 200° C = 473 K, with a volume of 0.5 m^3 is

$$n(473) \times 0.5 = 1.07 \times 10^{15} \tag{14}$$

Almost all of these are in the infrared, so we don't see the interior of an oven lit up.

Carroll & Ostlie show that the energy density for a blackbody is

$$u = aT^4 \tag{15}$$

where $a = 4\sigma/c = 7.565767 \times 10^{-16}$J m^{-3}K^{-4} is the radiation constant, and σ is the Stefan-Boltzmann constant. Using this, we can find the average energy per photon.

$$\langle E \rangle = \frac{u}{n} \tag{16}$$

$$= 3.73715 \times 10^{-23} T \tag{17}$$

$$= 2.707kT \tag{18}$$

Example 2. The average photon energy at the centre of the Sun, where $T=1.57\times10^{7}$ K, is

$$\langle E\rangle = 2.707\left(1.57\times10^{7}\right)k \tag{19}$$
$$= 5.87\times10^{-16}\text{ J} \tag{20}$$
$$= 3667\text{ eV} \tag{21}$$

In the Sun's atmosphere, $T=5777$ K and we have

$$\langle E\rangle = 2.707\,(5777)\,k \tag{22}$$
$$= 2.16\times10^{-19}\text{ J} \tag{23}$$
$$= 1.35\text{ eV} \tag{24}$$

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