RADIO ASTRONOMY - SPECTRAL FLUX DENSITY AND TELESCOPE EFFICIENCY

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Since radio waves are composed of photons with longer wavelengths, and thus lower energy, than visible light, the same analysis can be applied to radio telescopes as we used for optical telescopes. A single-dish radio telescope can be used to observe an object over a given range of frequencies (the *bandwidth*). Just as with optical telescopes, a sensitivity function gives the relative sensitivity of the telescope at different frequencies. The actual amount of radio energy received at a given frequency depends, therefore, on two things: the *spectral flux density* $S(\nu)$ of the radio source (the amount of energy per second, per unit frequency interval, received at the telescope from the source), and a filter function f_ν giving the efficiency of the telescope at various frequencies. The efficiency is a number between 0 and 1.

As you might expect, the energy received from an astronomical source is very small compared to terrestrial radio sources. The spectral flux density is measured in janskys (Jy, named after Karl Jansky, the originator of radio astronomy in the 1930s) where 1 Jy $=10^{-26}$ W m^{-2}Hz^{-1}.

Example. Suppose we have a 100 m diameter radio telescope with a triangular efficiency function. The detector has a bandwidth of 50 MHz centred at 1.430 GHz, so it is sensitive to frequencies between $\nu_\ell = 1.430-0.025 = 1.405$ GHz and $\nu_u = 1.430+0.025 = 1.455$ GHz, with a maximum sensitivity at $\nu_m = 1.430$ GHz. The function is therefore

$$f_\nu = \begin{cases} \frac{\nu}{\nu_m-\nu_\ell} - \frac{\nu_\ell}{\nu_m-\nu_\ell} = \frac{\nu-1.405}{0.025} & 1.405 \text{ GHz} \le \nu \le 1.430 \text{ GHz} \\ -\frac{\nu}{\nu_u-\nu_m} + \frac{\nu_u}{\nu_u-\nu_m} = \frac{-\nu+1.455}{0.025} & 1.430 \text{ GHz} \le \nu \le 1.455 \text{ GHz} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The sensitivity is 100% at the midpoint: $f_{1.430} = 1$.

For a radio source with a constant spectral flux density of $S = 2.5$ mJy $= 2.5\times10^{-29}$ W m^{-2}Hz^{-1} over the bandwidth, the total power measured by the telescope (with area $A = \pi\left(\frac{100}{2}\right)^2$) is

$$P_m = AS \int_{1.405\times 10^9}^{1.455\times 10^9} f_\nu d\nu \tag{2}$$

$$= \pi \left(\frac{100}{2}\right)^2 \left(2.5\times 10^{-29}\right) \left[\int_{1.405\times 10^9}^{1.430\times 10^9} \frac{\nu - 1.405\times 10^9}{0.025\times 10^9} d\nu + \int_{1.430\times 10^9}^{1.455\times 10^9} \frac{-\nu + 1.455\times 10^9}{0.025\times 10^9} d\nu\right] \tag{3}$$

$$= \pi \left(\frac{100}{2}\right)^2 \left(2.5\times 10^{-29}\right)\left(2.50\times 10^7\right) \tag{4}$$

$$= 4.91\times 10^{-18}\text{ W} \tag{5}$$

Radio telescopes receive an extremely small signal! However, given the enormous distance of the source, the emitted power is still substantial. If the source here is a galaxy at a distance of $d = 100$ Mpc, and it emits isotropically, then the total emitted power is:

$$P_e = 4\pi d^2 \left(2.5\times 10^{-29}\right)\left(2.50\times 10^7\right) \tag{6}$$

$$= 4\pi \left(100\times 10^6 \times 3.08567758\times 10^{16}\text{ m}\right)^2 \left(\left(2.5\times 10^{-29}\right)\left(2.50\times 10^7\right)\right) \tag{7}$$

$$= 7.48\times 10^{28}\text{ W} \tag{8}$$

This is roughly 200 times the total solar luminosity (summed over all wavelengths).