RELATIVISTIC ACCELERATION IN TERMS OF FORCE

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Ordinary force in relativity is given by

$$\mathbf{F}=\frac{m}{\sqrt{1-u^{2}/c^{2}}}\left[\mathbf{a}+\frac{(\mathbf{u}\cdot\mathbf{a})\,\mathbf{u}}{c^{2}-u^{2}}\right] \tag{1}$$

To get a general expression for the acceleration $\mathbf{a}$ in terms of the force, we take the dot product of both sides with the velocity $\mathbf{u}$:

$$\mathbf{u}\cdot\mathbf{F}=\gamma m\,(\mathbf{u}\cdot\mathbf{a})\left(1+\frac{u^{2}}{c^{2}-u^{2}}\right) \tag{2}$$

$$=\frac{\gamma mc^{2}\,(\mathbf{u}\cdot\mathbf{a})}{c^{2}-u^{2}} \tag{3}$$

$$=\gamma^{3}m\,(\mathbf{u}\cdot\mathbf{a}) \tag{4}$$

$$\mathbf{u}\cdot\mathbf{a}=\frac{\mathbf{u}\cdot\mathbf{F}}{\gamma^{3}m} \tag{5}$$

Substituting back into 1, we get

$$\mathbf{a}=\frac{\mathbf{F}}{\gamma m}-\frac{\mathbf{u}}{c^{2}-u^{2}}\frac{\mathbf{u}\cdot\mathbf{F}}{\gamma^{3}m} \tag{6}$$

$$=\frac{\mathbf{F}}{\gamma m}-\frac{\mathbf{u}}{\gamma mc^{2}}\,(\mathbf{u}\cdot\mathbf{F}) \tag{7}$$

In the limit of small $\mathbf{u}$, this reduces to the familiar Newton's law $\mathbf{F}=m\mathbf{a}$, but in the relativistic region, the acceleration depends on the object's velocity. As a result, the acceleration isn't parallel to the force unless $\mathbf{F}$ is either parallel to $\mathbf{u}$ or $\mathbf{F}\perp\mathbf{u}$; in the latter case $\mathbf{u}\cdot\mathbf{F}=0$ and the second term is zero.

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