ROSSELAND MEAN OPACITY

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Post date: 10 Jul 2023.

The opacity of a substance such as the atmosphere of a star or planet is a measure of how much absorption and/or scattering of photons occurs in the medium. The opacity is given the symbol κ_λ with the subscript λ indicating that in general, the opacity depends on the wavelength of the light or other radiation.

The opacity is due to a number of causes, each with its own value. The main ones are

(1) **Bound-bound transitions in atoms**. If a photon hits an atom (typically hydrogen or helium) with one or more electrons in a bound state, the photon can be absorbed by an electron and bump it up into a higher, excited state. This occurs if the photon just matches the energy difference between the electron's two states. The opacity here is denoted by κ_{bb}. (I'll drop the λ subscript from here on to avoid clutter.)

(2) **Bound-free transitions.** If a photon's energy is equal to or greater than the energy required to completely remove an electron from an atom, it can be absorbed by the electron. Any excess of the photon's energy over the ionization energy is given to the electron as kinetic energy. This absorption causes the bound-free opacity κ_{bf}.

(3) **Free-free absorption.** Although a totally free electron cannot absorb a photon due to energy conservation constraints, a free electron in the vicinity of another particle such as an ion can absorb a photon, and thus increase its kinetic energy. Viewed in the centre of momentum frame, the extra ion is needed in order that energy be conserved in the interaction. This opacity is κ_{ff}.

(4) **Electron-photon scattering.** Rather than being absorbed, a photon can scatter off an electron and thus get redirected away from its initial path. One such process is Compton scattering. Typically there is some energy exchange between the photon and electron, so that the outgoing photon's wavelength is different. The opacity due to scattering is κ_{es}.

(5) **Hydrogen ion scattering.** A hydrogen atom is capable of binding an additional electron, albeit weakly, giving the negative hydrogen ion H^-. The ionization energy of this extra electron is only 0.754 eV, compared to the 13.6 eV for the 'normal' electron, so a hydrogen ion can absorb fairly low energy photons. This opacity is κ_{H^-}.

As mentioned, all of these opacities depend on the wavelength of the photons. In practice, an average opacity, with the average taken over wavelength (or frequency) is given by the *Rosseland mean opacity* $\overline{\kappa}$. The sources of opacity listed above are independent, so the total opacity is the sum of all of them. Thus the Rosseland mean opacity can be written as

$$\overline{\kappa}=\overline{\kappa}_{bb}+\overline{\kappa}_{bf}+\overline{\kappa}_{ff}+\overline{\kappa}_{es}+\overline{\kappa}_{H^-} \tag{1}$$

The bound-bound transitions are complex affairs, and there is no tractable equation for $\overline{\kappa}_{bb}$. Approximations for the other terms do exist, however. These are given in Carroll & Ostlie's Chapter 9, and are

$$\overline{\kappa}_{bf}=4.34\times10^{21}\frac{g_{bf}}{t}Z\left(1+X\right)\frac{\rho}{T^{3.5}}\text{ m}^2\text{kg}^{-1} \tag{2}$$

$$\overline{\kappa}_{ff}=3.68\times10^{18}g_{ff}\left(1-Z\right)\left(1+X\right)\frac{\rho}{T^{3.5}}\text{ m}^2\text{kg}^{-1} \tag{3}$$

$$\overline{\kappa}_{es}=0.02\left(1+X\right)\text{ m}^2\text{kg}^{-1} \tag{4}$$

$$\overline{\kappa}_{H^-}=7.9\times10^{-34}\frac{Z}{0.02}\sqrt{\rho}T^9\text{ m}^2\text{kg}^{-1} \tag{5}$$

where ρ is the mass density and T is the temperature, as usual.

There are a number of parameters in these equations. The factors g_{bf} and g_{ff} are *Gaunt factors* and are derived from quantum mechanical considerations. For visible and ultraviolet light, both of them are approximately 1. The factor t is called the *guillotine factor*, and accounts for the fact that an atom cannot contribute to bound-free opacity after it has been ionized. Typical values for t range from 1 to 100.

The parameter X is the fraction (by mass) of gas that is hydrogen, and Z is the fraction of gas that is not either hydrogen or helium. Astrophysicists refer to all elements other than hydrogen or helium as 'metals', which can be a bit confusing for novices. [There is also a symbol Y for the fraction of helium, but it doesn't appear in these equations.] Thus $X+Y+Z=1$.

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