THE LIGHT CLOCK AND TIME DILATION

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One of the original thought experiments that led to the equation of time dilation is the *light clock*. In a railway carriage a light source is placed on one side of the carriage and a mirror on the wall directly opposite the light. The line between the light and mirror is perpendicular to the direction of the train's motion, so that the light beam travels across the car rather than along it. To an observer in the train (frame S'), the time taken for the light to make a round trip to the mirror and back is

$$\Delta t' = \frac{2d}{c} \tag{1}$$

where d is the width of the carriage.

The train now moves at speed u relative to an observer standing beside the track (in frame S). This observer sees the light follow a diagonal path across the carriage and back, but the speed of the light is still c. How long Δt does S say that the light takes to make the round trip?

Since lengths perpendicular to the motion are unaffected, S says that the width of the carriage is still d. In time Δt the train travels a distance $u\Delta t$ in frame S and since the light takes the same time on each leg of its journey, it reaches the opposite wall in time $\Delta t/2$ during which time the train has travelled a distance $u\Delta t/2$. The path of the light beam is therefore the hypotenuse of a right-angled triangle with sides of lengths d across the train and $u\Delta t/2$ along it. The component of the light's velocity perpendicular to the direction of motion is therefore

$$u_{\perp} = c\frac{d}{\sqrt{d^2 + (u\Delta t/2)^2}} \tag{2}$$

so the time taken to cross the train once is

$$\frac{\Delta t}{2} = \frac{d}{u_{\perp}} \tag{3}$$

$$= \frac{1}{c}\sqrt{d^2 + (u\Delta t/2)^2} \tag{4}$$

Solving for Δt we get

$$\Delta t = \frac{2d}{c}\frac{1}{\sqrt{1-u^2/c^2}} = \gamma\Delta t' \tag{5}$$

That is, the time measured by S is longer than that measured by S' by the factor γ, so that S thinks that the clock in frame S' runs slow. This is the time dilation effect.