UNCERTAINTY PRINCIPLE - VISUALIZATION WITH FOURIER SERIES

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog and include the title or URL of this post in your comment.

Post date: 15 Jun 2023.

To represent a real free particle, we need to write its wave function as the superposition of plane waves of different wavelengths, in order that the overall wave function is normalizable. In general, we need to use a Fourier transform to do this (that is, we need to integrate over a continuous range of wavelengths). However, we can get a feel for the procedure by using a Fourier series instead, in which we sum over a finite number of discrete wavelengths.

We'll have a look at the series:

$$\Psi = \frac{2}{N+1}\left[\sin x - \sin 3x + \sin 5x - \ldots \pm \sin Nx\right] \tag{1}$$

$$= \frac{2}{N+1}\sum_{n=1,odd}^{N}(-1)^{(n-1)/2}\sin nx \tag{2}$$

This defines a wave packet in the interval $x \in [0,\pi]$ which peaks at $x = \pi/2$. Using Maple, we can generate plots of Ψ for various values of N, shown in Figs 1 through 4.

If we define the width of the central peak as the range Δx between the values of x for which $\Psi = 0.5$, we can use Maple's fsolve command to find these values of x (see Table 1).

The location of a particle represented by the wave function Ψ is more accurately known for higher N. Conversely, the momentum is better known

N	Δx
5	0.638
11	0.317
21	0.172
41	0.090

TABLE 1. Width of central peak.

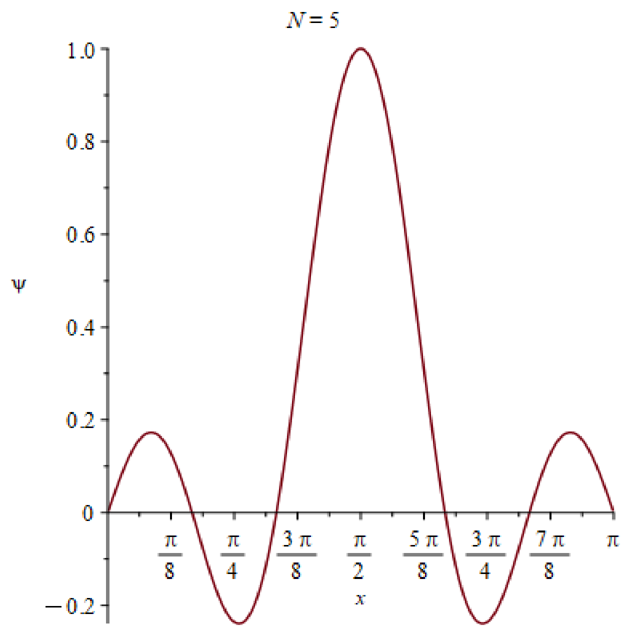

FIGURE 1. Fourier series with $N = 5$.

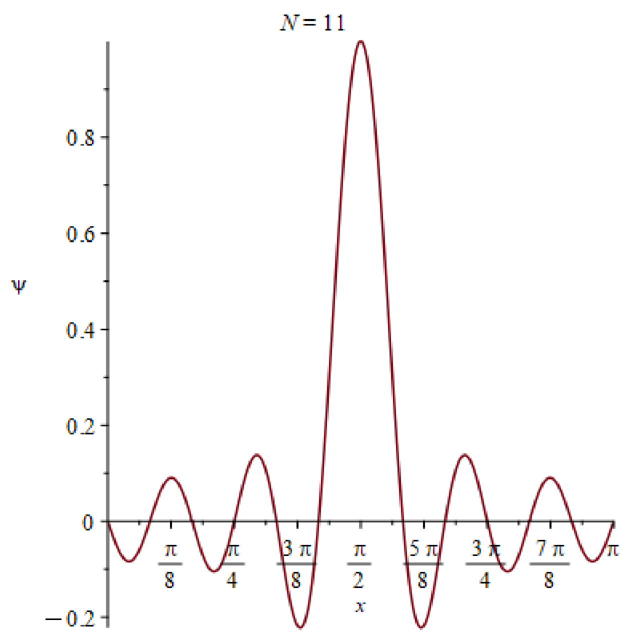

FIGURE 2. Fourier series with $N = 11$.

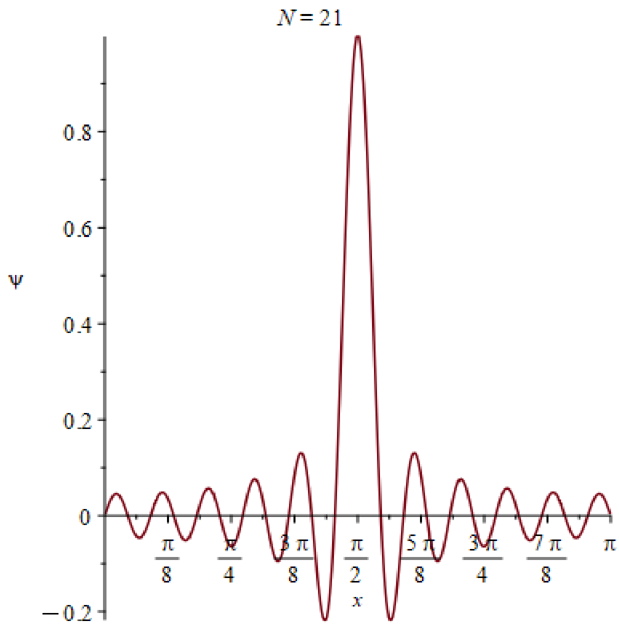

FIGURE 3. Fourier series with $N = 21$.

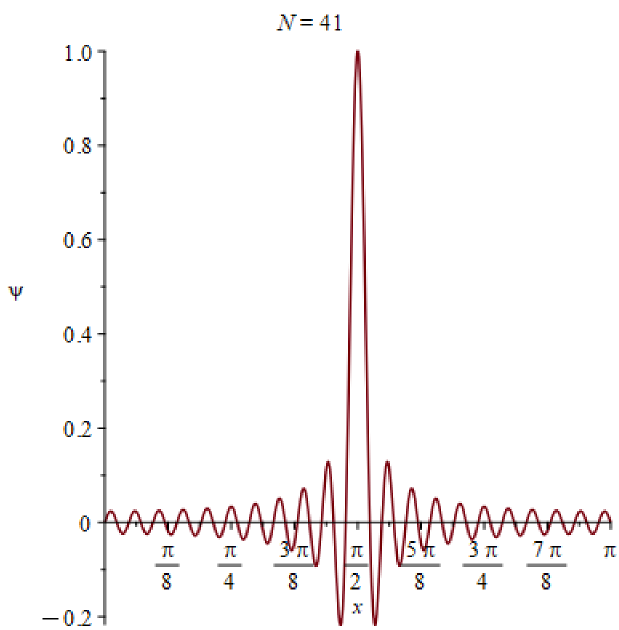

FIGURE 4. Fourier series with $N = 41$.

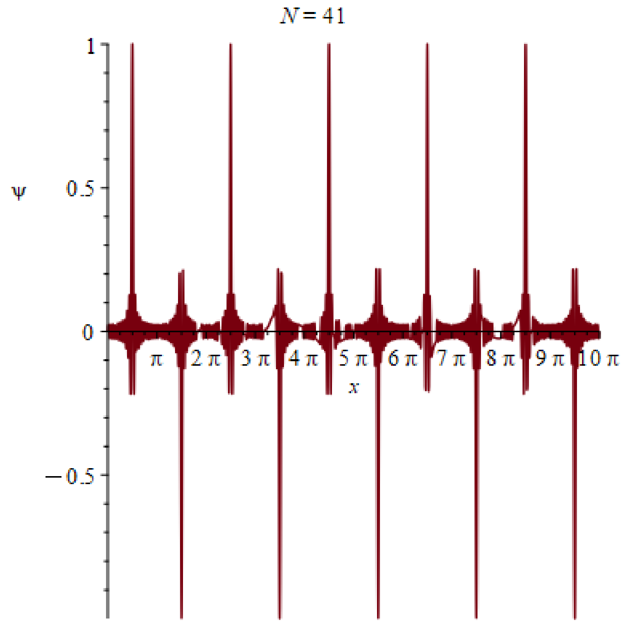

FIGURE 5. Fourier series for larger range of x.

for lower N, since there are fewer wavelengths (hence, fewer energies and momenta) contributing to Ψ if N is smaller. This is a graphic representation of the position-momentum uncertainty principle.

By the way, the equation 2 defines a wave packet only for the region $0 \leq x \leq \pi$. If we use this equation for a larger interval, say $0 \leq x \leq 10\pi$, we get Fig. 5.

To define a single wave packet over an interval $0 \leq x \leq A\pi$ we need to modify 2 like this:

$$\Psi = \frac{2}{N+1} \sum_{n=1,odd}^{N} (-1)^{(n-1)/2} \sin \frac{nx}{A} \tag{3}$$