SIDEREAL AND SYNODIC PERIODS OF PLANETS

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 1, Problems 1.1 - 1.3.

As I recently got a shiny new telescope (for looking at the night sky, not the neighbours!), I thought it would be a good idea to revisit some astrophysics since it's been something like 40 years since I studied it properly. The book I've chosen for this is the one referenced above, and it will take a while for me to work through it, since it has 30 chapters and comes close to 1500 pages. However, since I live in Scotland where clear nights are rare and the sky never gets truly dark between May and July (due to being so far north; the town of Monifieth where I live is almost as far north as Juneau, Alaska), having a form of astronomy that doesn't depend on actually looking at the sky is very useful.

So let's begin with some basics about planetary orbits. For most of history, people thought that the Earth was the centre of the universe. Looking at the sky, particularly at night, reinforced this view since all objects visible in the sky appear to rotate around the Earth and since the Earth felt solid an immovable (except during earthquakes), it was just common sense that the Earth was fixed and everything else revolved around it. This form of common sense is the same kind that got Aristotle in trouble by stating that the natural state of objects was at rest (since everything on Earth seemed to come to rest, given long enough) and that heavier objects fell faster than lighter ones (well, a cannonball falls faster than a feather anyway).

In 1543, Polish astronomer Nicolaus Copernicus published *On the Revolution of the Celestial Sphere*, in which he proposed that the Sun, not the Earth, was the true centre of the universe. Copernicus was still a bit of a mystic, however, so he assumed that the orbits of everything around the Sun were all geometrically 'perfect' circles. Since the planets' orbits are all elliptical to some extent, this severely limited Copernicus's theory's ability to predict planetary positions, but it was still a giant leap forward in astronomical thought.

One consequence of the Copernican theory is that it is relatively easy to calculate the synodic period of a planet if we know its sidereal period. First, we need to define these two terms.

The *sidereal period* of a planet is simply the time it takes to make one complete orbit around the Sun. Thus Earth's sidereal period is 1 year.

To understand the concept of a *synodic period*, we need to think about some observational astronomy. First, consider a *superior planet* (that is, a planet whose orbit is further from the Sun than Earth's, such as Mars). When the positions of Earth and Mars are just right, Mars is directly opposite the Sun in the sky (that is, the angle between a line from Earth to Sun and a line from Earth to Mars is π or $180°$). When this happens, Mars is said to be in *opposition*. After opposition, both planets continue in their orbits until some future time at which they line up again, giving another opposition. The time between two successive oppositions is the synodic period.

For *inferior planets* (planets whose orbits are closer to the Sun than Earth's, of which there are only two: Venus and Mercury), opposition can never be achieved since it's impossible for the Earth to get between the Sun and the planet. However, the Earth, the inferior planet and the Sun can still line up in two different ways. When the planet is directly between the Earth and the Sun, it is in *inferior conjunction*, at which point it's at its closest position to Earth. When the planet is directly on the other side of the Sun, it is in *superior conjunction*, at which point it is at its furthest position from Earth. (Superior planets can be in superior conjunction as well, of course, but never in inferior conjunction. As a result, superior conjunction for a superior planet is usually called just 'conjunction'.)

When Venus or Mercury is in inferior conjuction with the Earth, the Earth is in opposition as seen from Venus or Mercury.

We can work out the relation between the sidereal and synodic periods using a bit of geometry. Consider a superior planet such as Mars, and let's suppose that at $t=0$, Mars is in opposition. How long will it be before the next opposition? Since Mars takes longer to orbit the Sun, its angular speed ω_M is smaller than that of Earth ω_E. After a synodic period S has elapsed, the position angles of Earth and Mars must be equal again, but since Earth is moving faster, it will have made one more complete orbit around the Sun than Mars has. The angle swept out by Earth in time S is $\omega_E S$, so

$$\omega_E S = \omega_M S + 2\pi \tag{1}$$

In terms of the sidereal period of Earth $\omega_E = 2\pi/P_E$ (with a similar relation for Mars), so

$$\frac{2\pi}{P_E}S = \frac{2\pi}{P_M}S + 2\pi \tag{2}$$
$$\frac{1}{S} = \frac{1}{P_E} - \frac{1}{P_M} \tag{3}$$

If all times are in years, then for any superior planet with period P

$$\frac{1}{S} = 1 - \frac{1}{P} \tag{4}$$

For an inferior planet, the roles of Earth and the planet are reversed, so

$$\frac{1}{S} = \frac{1}{P} - 1 \tag{5}$$

All of this, of course, assumes perfectly circular orbits with planets moving at constant speeds, so the formulas aren't exact. However, we can get an idea of how good they are by plugging in some actual numbers.

Planet	P (years)	S (calc)	S (actual)
Mercury	0.241	0.3175	0.3176
Venus	0.616	1.604	1.599
Mars	1.9	2.111	2.136
Jupiter	11.9	1.092	1.092
Saturn	29.5	1.035	1.035
Uranus	84.0	1.012	1.013
Neptune	164.8	1.0061	1.0075
Pluto	248.5	1.0040	1.0048

The agreement is suprisingly good for such a simple formula. As you might expect, the further out the planet is, the closer S gets to 1 Earth year, since the outer planets don't move that much in their own orbits over the course of a year. Thus the shortest synodic period for a superior planet is for the planet that is furthest away, namely Pluto (or, if you insist on degrading Pluto to the status of a dwarf planet, Neptune). For amateur astronomers, this means that each superior planet (Jupiter and beyond) is best placed for observing (which happens when it's at opposition) roughly once per year. Mars is at closest approach to Earth about every two years.

Historically, the synodic period of a planet would be the available datum for the planet (since it's easy to measure the time from one opposition or conjunction to the next), and the formula above would be inverted to give the planet's actual period as

$$P = \begin{cases} \left(1 - \frac{1}{S}\right)^{-1} & \text{superior planet} \\ \left(1 + \frac{1}{S}\right)^{-1} & \text{inferior planet} \end{cases} \tag{6}$$

Given the observational data available to someone, such as Copernicus, in the pre-telescope age, we can work out the relative ordering of the planets from the Sun. Since Mercury and Venus are never observed in opposition, they must have orbits closer to the Sun than Earth, and since Mercury's greatest elongation (angular separation from the Sun) is less than that of Venus, we can deduce that Mercury must be closer to the Sun than Venus.

For the superior planets, we can order them according to decreasing synodic period, which gives the correct ordering of Mars through Pluto (or up to Saturn, since Uranus, Neptune and Pluto were discovered only after the invention of the telescope).

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