ESCAPE SPEED

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 2, Problem 2.7.

The total energy of an object of mass m at a distance of r from the centre of mass of another body of mass M is the sum of its kinetic and gravitational potential energies, which are

$$K = \frac{1}{2}mv^2 \tag{1}$$

$$V = -\frac{GMm}{r} \tag{2}$$

The escape speed from M is defined as the speed an object must be travelling directly away from the mass M in order for it to have zero speed when it reaches infinity. Only if m is capable of getting arbitrarily far away from M can it be said to have escaped from M, since if it comes to rest at a finite distance from M, the gravitational attraction between m and M will accelerate m back towards M.

If $v = 0$ when $r = \infty$, then at infinity $K = V = 0$ and since total energy $E = K + V$ is conserved, $E = 0$ at all points along m's path away from M. If m starts at a distance r_0 from M then if m is to escape, it must have a speed v_0 given by

$$\frac{1}{2}mv_0^2 = \frac{GMm}{r_0} \tag{3}$$

$$v_0 = \sqrt{\frac{2GM}{r_0}} \tag{4}$$

The mass m drops out of the formula for v_0, so *any* mass travelling away from M with speed v_0 at $r = r_0$ will escape.

Example 1. The escape speed from the surface of Jupiter is found from Jupiter's mass and radius, which are

$$M_J = 1.898 \times 10^{27} \text{ kg} \quad (5)$$

$$r_J = 6.9911 \times 10^{7} \text{ m} \quad (6)$$

Therefore

$$v_J = 6.018 \times 10^{4} \text{ m s}^{-1} \approx 60 \text{ km s}^{-1} \quad (7)$$

This compares with v_0 from the surface of Earth which is $v_0 = 11.2$ km s^{-1}. Note, by the way, that this does *not* mean that rockets that are fired into space from Earth must go at 11.2 km s^{-1}. A rocket has a steady force generated by its engines as it leaves the launch pad, so the only requirement is that the engines produce a force upwards that is greater than the mg force due to gravity. In principle, the rocket can be travelling at any speed, no matter how slowly, and still reach orbit as long as the engines provide slightly more than mg upwards.

Example 2. If an object starts from the Earth's orbit (but at a point away from the Earth, so that Earth's gravitational force is negligible), we can find the escape speed from the solar system. We need the mass M_S of the Sun and the radius r_E of the Earth's orbit, which are

$$M_S = 1.989 \times 10^{30} \text{ kg} \quad (8)$$

$$r_E = 1 \text{ AU} = 1.496 \times 10^{11} \text{ m} \quad (9)$$

Therefore

$$v_E = 42.1 \text{ km s}^{-1} \quad (10)$$