KEPLER'S THIRD LAW AND SATELLITE ORBITS

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 2, Problem 2.8.

From Kepler's third law, we can work out some facts about satellite orbits around the Earth. The law is

$$P^2 = \frac{4\pi^2}{GM} a^3 \tag{1}$$

where P is the period, a is the semimajor axis and M is the total mass of the system (effectively, the mass of the Earth when dealing with satellites).

Example 1. The Hubble Space Telescope is in a nearly circular orbit about 610 km above the surface of the Earth. The mean radius of the Earth is 6.371×10^6 m and its mass is 5.972×10^{24} kg so we have

$$a = 6.371 \times 10^6 + 6.1 \times 10^5 = 6.981 \times 10^6 \text{ m} \tag{2}$$

$$P = \sqrt{\frac{4\pi^2 \left(6.981 \times 10^6\right)^3}{\left(6.67 \times 10^{-11}\right)\left(5.972 \times 10^{24}\right)}} = 5807 \text{ s} = 1.61 \text{ hr} \tag{3}$$

Example 2. A geosynchronous orbit is an orbit that keeps a satellite directly over a fixed spot on the Earth's surface. Such an orbit is possible only over the equator, since the centre of the orbit has to be the centre of the Earth and the orbit has to be parallel to a plane of latitude.

The period of a geosynchronous orbit must be exactly one sidereal day (*not* a solar day of 24 hours), since the Earth rotates a little more than 2π in a solar day in order for a fixed point on the Earth to face the Sun at the same time in the day. A sidereal day is shorter by about 4 minutes than a solar day, so in this case

$$P = 23h56m04s = 86164 \text{ s} \tag{4}$$

The semimajor axis of such an orbit is then

$$
\begin{aligned}
a &= \left[\frac{GMP^2}{4\pi^2}\right]^{1/3} \qquad (5)\\
&= 4.215 \times 10^7 \text{ m} = 42,150 \text{ km} \qquad (6)
\end{aligned}
$$

This is about 6 times the radius of the Earth. Due to the large height of these satellites, they are visible from virtually anywhere on the hemisphere centred at the point over which the satellite orbits the Earth, so even though they must orbit above the equator, they can be used for communication, GPS and so on from most of the Earth.