KEPLER'S THIRD LAW AND THE MOONS OF JUPITER

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 2, Problem 2.12.

We can check Kepler's third law for Jupiter's Galilean moons. The law states that

$$P^2 = \frac{4\pi^2}{GM}a^3 \tag{1}$$

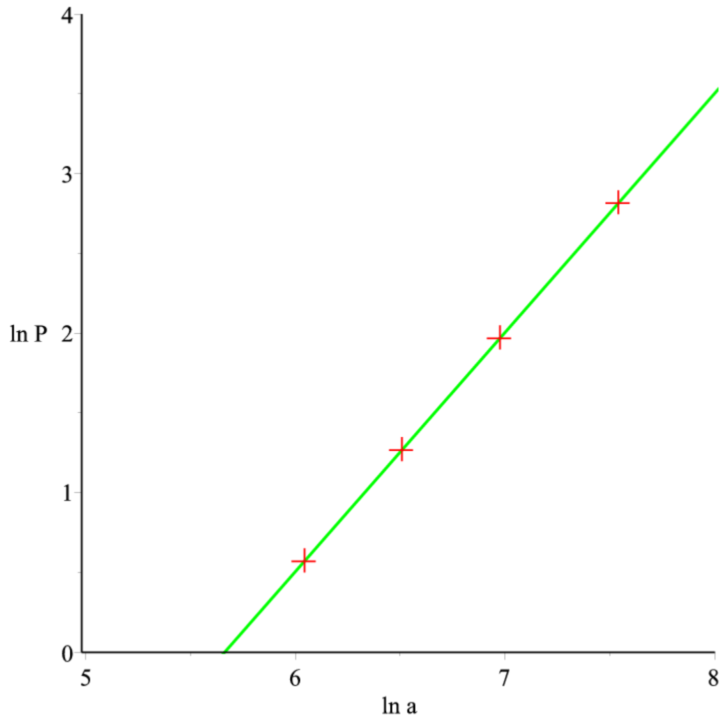
Using data from Carroll & Ostlie's Appendix C, we have

	P (days)	a $\left(10^3 \text{ km}\right)$
Io	1.769	421.6
Europa	3.551	670.9
Ganymede	7.155	1070.4
Callisto	16.689	1882.7

We can plot $\ln P$ versus $\ln a$ and then do a least squares fit of a straight line to the four points. From theory, we should have

$$\ln P = \frac{1}{2}\ln\left(\frac{4\pi^2}{GM}\right) + \frac{3}{2}\ln a \tag{2}$$

so the slope of the line should be 1.5. Here's the plot:



The red crosses are the data points and the green line is the best fit straight line. The least squares fit (using Maple's Statistics[LeastSquares] function) gives

$$\ln P = -8.494 + 1.5 \ln a \tag{3}$$

so the slope is indeed 1.5.

From the y-intercept, we have

$$\frac{1}{2} \ln \left(\frac{4\pi^2}{GM} \right) = -8.494 \tag{4}$$

where M is the mass of Jupiter (plus the mass of the satellite, which we can ignore as it's negligible compared to Jupiter). To get the mass of Jupiter, however, we need to get the units right since we're using days instead of seconds and thousands of kilometres instead of metres. We can convert the data to SI units, but it's easier to just convert G. The SI value of G is 6.67384×10^{-11} $\mathrm{m^3 kg^{-1} s^{-2}}$ so in our units we have

(5)

$$G = 6.67384 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}} \times \left(10^{-6}\left[10^3\ \mathrm{km/m}\right]\right)^3 \times (3600 \times 24\ \mathrm{sec/day})^2$$

(6)

$$= 4.982 \times 10^{-19}\left[10^3\ \mathrm{km}\right]^3 \mathrm{kg^{-1} day^{-2}}$$

Therefore

$$M_J = \frac{4\pi^2}{Ge^{-2\times 8.494}} \tag{7}$$

$$= 1.893 \times 10^{27}\ \mathrm{kg} \tag{8}$$

The actual value is 1.898×10^{27} kg so this simple calculation is quite close.