HALLEY'S COMET: AN APPLICATION OF KEPLER'S LAWS

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 2, Problem 2.14.

We can use Kepler's laws to work out a few facts about Halley's comet. Given that its period is $P = 76$ years and its eccentricity is $e = 0.9673$, we can find the semimajor axis a by comparing the comet's orbit to Earth's orbit, where $a = 1$ AU and $P = 1$ year. From Kepler's third law (and ignoring the masses of the comet and Earth relative to that of the Sun):

$$\frac{P_H^2}{P_E^2} = \frac{a_H^3}{a_E^3} \tag{1}$$

$$a_H = P_H^{2/3} = 17.94 \text{ AU} \tag{2}$$

Also from the third law, we can obtain an estimate of the Sun's mass:

$$P_H^2 = \frac{4\pi^2}{GM} a_H^3 \tag{3}$$

$$M = \frac{4\pi^2}{GP_H^2} a_H^3 \tag{4}$$

In SI units

$$P_H = 76 \text{ years} = 2.4 \times 10^9 \text{ s} \tag{5}$$

$$a_H = 17.94 \text{ AU} = 17.94 \times \left(1.496 \times 10^{11} \text{ m AU}^{-1}\right) = 2.68 \times 10^{12} \text{ m} \tag{6}$$

This gives

$$M = \frac{4\pi^2 \left(2.68 \times 10^{12}\right)^3}{\left(6.67 \times 10^{-11}\right)\left(2.4 \times 10^9\right)^2} \tag{7}$$

$$= 1.986 \times 10^{30} \text{ kg} \tag{8}$$

The currently accepted mass of the Sun is 1.989×10^{30} kg.

The perihelion and aphelion distances are

$$r_p = a(1-e) = 0.587 \text{ AU} \tag{9}$$
$$r_a = a(1+e) = 35.29 \text{ AU} \tag{10}$$

The comet gets closer to the Sun than Venus and at aphelion it is further away than Neptune.

In accordance with Kepler's second law (equal areas in equal times), its speed varies considerably over its orbit. The speed is given by

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \tag{11}$$

so at perihelion and aphelion we have

$$v_p = \sqrt{\frac{GM}{a}\left(\frac{2}{1-e} - 1\right)} \tag{12}$$
$$= \sqrt{\frac{(6.67\times10^{-11})(1.989\times10^{30})}{2.68\times10^{12}}\left(\frac{2}{1-0.9673} - 1\right)} \tag{13}$$
$$= 5.457\times10^{4} \text{ m s}^{-1} \tag{14}$$
$$= 54.57 \text{ km s}^{-1} \tag{15}$$
$$v_a = \sqrt{\frac{(6.67\times10^{-11})(1.989\times10^{30})}{2.68\times10^{12}}\left(\frac{2}{1+0.9673} - 1\right)} \tag{16}$$
$$= 907 \text{ m s}^{-1} \tag{17}$$

On the semiminor axis, from the properties of an ellipse

$$r^2 = a^2e^2 + b^2 \tag{18}$$
$$= a^2e^2 + a^2\left(1-e^2\right) \tag{19}$$
$$= a^2 \tag{20}$$
$$r = a \tag{21}$$

so the speed at these two points is

$$v_b = \sqrt{\frac{GM}{a}} = \sqrt{\frac{(6.67\times10^{-11})(1.989\times10^{30})}{2.68\times10^{12}}} = 7.04 \text{ km s}^{-1} \tag{22}$$

The ratio of kinetic energies at perihelion and aphelion is therefore

$$\frac{v_p^2}{v_a^2} = \left(\frac{5.457 \times 10^4}{907}\right)^2 = 3620 \tag{23}$$