MARS'S ORBIT: ELLIPSE VERSUS CIRCLE

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 2, Problem 2.17.

In his early attempts to figure out the orbits of the planets, Kepler tried to fit the data using only circles for the orbits. One way of improving the agreement is to shift the centre of the circle away from the centre of mass (or just the Sun, in Kepler's case). To get an idea of good this approximation is, we can look at the orbit of Mars.

Mars is a good test case because its eccentricity of 0.0934 is the second largest (after Mercury), so if we can get a good approximation of its orbit with a circle, such approximations should also work well for the other planets with lower eccentricities.

We can use the Maple program from the last post to calculate the true (elliptical) orbit of Mars, given its eccentricity of $e = 0.0934$ and semimajor axis $a = 1.524$ AU. To compare this orbit to the best fit of a circle, I took the approximating circle to have the same centre as the ellipse (note that the centre of the ellipse is *not* the same thing as its focus) and a radius that is the average of the semimajor and semiminor axes a and b of the ellipse. To plot this circle on the same polar plot as the ellipse, we need the equation of this circle in polar form. The origin of the polar coordinate system is taken to be the principal focus of the ellipse and since the distance from the focus to the centre is ae (see the properties of ellipses), the centre of both the ellipse and circle is at $(x,y) = (-ae, 0)$, so the equation of the circle in rectangular coordinates is

$$(x+ae)^2 + y^2 = \left(\frac{a+b}{2}\right)^2 \equiv \beta^2 \tag{1}$$

To convert to polar coordinates, we use

$$x = r\cos\theta \tag{2}$$

$$y = r\sin\theta \tag{3}$$

which gives the quadratic equation

$$r^2 + 2aer\cos\theta + a^2e^2 - \beta^2 = 0 \tag{4}$$

The positive (since we must have $r \geq 0$) solution is

$$\begin{aligned} r &= \sqrt{a^2e^2\cos^2\theta - a^2e^2 + \beta^2} - ae\cos\theta & (5)\\ &= \sqrt{\beta^2 - a^2e^2\sin^2\theta} - ae\cos\theta & (6) \end{aligned}$$

The semiminor axis is given by

$$b = a\sqrt{1-e^2} \tag{7}$$

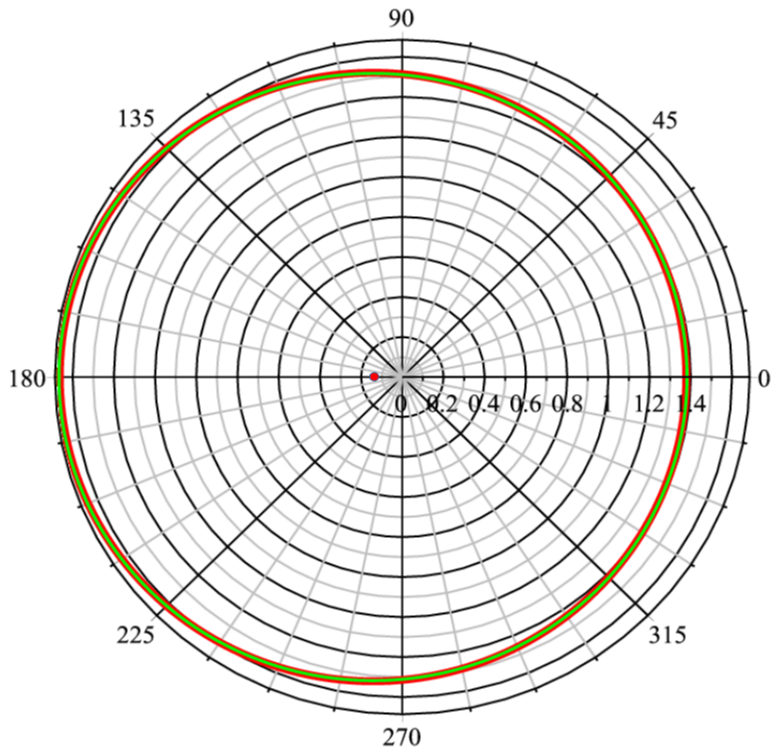
so the radius of the circle is

$$\beta = \frac{a}{2}\left(1 + \sqrt{1-e^2}\right) \tag{8}$$

A Maple procedure for generating a polar plot of the circle is

```
with(plots):
rad2deg := 180/Pi;
circ := proc (a, e)
local beta;
beta := (1/2)*(1+sqrt(1-e^2))*a; polarplot(sqrt(beta^2-a^2*e^2*sin(theta/rad2deg)^2)-a
  theta = 0 .. 360, angularunit = degrees,
  color = "red", thickness = 4)
end proc
ellipse0 := orbit(1, 1.524, 0.934e-1, 1000, "green")
display([circ(1.524, 0.934e-1), ellipse0])
```

The plot *ellipse0* is generated using the *orbit* procedure from the previous post. The result is as follows:



The red dot just to the left of the origin shows the centre of the circle (and ellipse), with the focus of the ellipse at the origin, and we can see that the circle (in red) and the ellipse (green) are almost exactly the same. Thus it's not surprising that Kepler was able to obtain good agreement by using offset circles, and it's to his credit that he didn't just ignore the small discrepancies (only 8' of arc) between the predictions of this model and Tycho Brahe's observations. His persistence led to his three laws and the discovery that the true orbit is an ellipse.