PARALLAX

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 3, Problem 3.1.

One of the biggest problems in astronomy is the determination of the distance to a celestial object. There are various ways of doing this, but for objects that are relatively nearby, *parallax* is one commonly used method. Suppose, for example, that we want to determine the distance to Mars when Mars is at its closest approach to Earth (in opposition). If we take simulataneous measurements of the the position of Mars on opposite sides of the Earth, then the position of Mars together with the positions of the two measurements form an isosceles triangle with a base length equal to the diameter of the Earth and a height equal to the distance to Mars. If we drop a line from Mars down to the base so that this line bisects the base, then the angle θ that this line makes with either long side of the triangle has the property that

$$\tan\theta = \frac{R}{d} \tag{1}$$

where R is the radius of the Earth and d is the distance to Mars. We can measure θ by comparing the positions of Mars as seen on either side of the Earth with background stars which will not show any appreciable parallax, and thus determine d. For small angles (as any parallax of a celestial object will be), $\tan\theta \approx \theta$ so we have

$$d = \frac{R}{\theta} \tag{2}$$

where θ is measured in radians.

In 1672, an expedition was mounted to make such a measurement, with the result that the angular position of Mars as measured on opposite sides of the Earth was 33.6 arc seconds, which is 1.629×10^{-4} radians. This angle is twice θ in the above formulas (since it is measured from either end of the base of the isosceles triangle), so the distance to Mars from this measurement is

$$d=\frac{R}{\theta/2}=\frac{2\times6371}{1.629\times10^{-4}}=7.822\times10^{7}\text{ km}=0.523\text{ AU} \tag{3}$$

In 1672, however, synchronizing clocks situated on opposite sides of the Earth would have been a major problem. Suppose we wanted the error in d to be under 10%; how much of a time difference Δt could we have between the two clocks? For the purposes of this estimate, we'll neglect the rotation of the Earth and assume that Earth and Mars move in parallel straight lines over the time Δt. The average orbital velocities of Earth and Mars are $v_E = 29.79\text{ km s}^{-1}$ and $v_M = 24.13\text{ km s}^{-1}$, respectively.

The speed of Earth relative to Mars is therefore $v_E - v_M$, so the effective base of the triangle becomes $2R+(v_E-v_M)\Delta t$ so the distance to Mars is measured as

$$d'=\frac{2R+(v_E-v_M)\Delta t}{2\theta} \tag{4}$$

If we want the error to be less than 10%, we want $(d'-d)/d<0.1$ so

$$\frac{(v_E-v_M)\Delta t/2\theta}{R/\theta} < 0.1 \tag{5}$$

$$\Delta t < 0.1\frac{2R}{(v_E-v_M)} \tag{6}$$

$$\Delta t < 225\text{ sec} \tag{7}$$

Thus the clocks need to be synchronized to a little better than 4 minutes to get 10% accuracy.

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