BLACKBODY RADIATION RATE

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 3, Problem 3.8.

Although stars are, in general, not black (far from it), their radiation spectrum is in many cases closely approximated by that of a blackbody. In our earlier derivation from quantum mechanics, we found that the energy density of a blackbody is

$$\overline{\rho}(\lambda) = \frac{16\pi^2 c\hbar}{\lambda^5}\frac{1}{e^{A/\lambda}-1} \tag{0.1}$$

$$A = \frac{2\pi c\hbar}{k_B T} = \frac{hc}{k_B T} \tag{0.2}$$

The units of $\bar{\rho}$ are energy per unit volume. In order to get formula 3.22 in Carroll and Ostlie, we need to think about what this radiation is doing. The individual photons within the volume are moving in random directions with speed c, so if we want the rate per unit area at which energy is being transported in a given direction, we need first to multiply $\bar{\rho}$ by the speed of propagation c. That gives the total rate per unit area at which energy is being transported out of the unit volume. To get the amount in some specified direction, what we really want is the rate of radiation per unit solid angle. Since the solid angle of a sphere is 4π, we need to divide by 4π. This gives the rate at which energy is transported per unit time (the power) per unit area, per unit wavelength interval, per steradian.

$$B_\lambda(T) = \frac{\bar{\rho}c}{4\pi} \tag{0.3}$$

$$= \frac{4\pi c^2\hbar}{\lambda^5}\frac{1}{e^{A/\lambda}-1} \tag{0.4}$$

$$= \frac{2hc^2}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)} \tag{0.5}$$

The total rate of radiation per unit area, integrated over all wavelengths, is (using Maple)

$$\int_0^\infty B_\lambda(T)\,d\lambda = \frac{2\pi^4 k_B^4}{15h^3c^2}T^4 \tag{0.6}$$

To get the total rate of radiation per unit area, over all directions, for a surface of a finite body we consider only half the total solid angle, since the radiation away from the body will go only outwards. The radiation B_λ itself doesn't depend on angle, but when integrating over the polar angle θ we'll need to include a factor of $\cos\theta$. To see this, suppose we have a flat unit area in the xy plane. Radiation leaving this unit area in the z direction has maximum intensity. Radiation that leaves the unit area at an angle θ with the z axis has its intensity per unit area on a plane perpendicular to its propagation direction reduced by a factor of $\cos\theta$ (or, looked at another way, in order to receive the full intensity in this direction, we'd need to include radiation from an area of $1/\cos\theta$). Therefore, the solid angle integral is

$$\int \cos\theta\,d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta\;\sin\theta\cos\theta = \pi \tag{0.7}$$

The power j emitted per unit area is the Stefan-Boltzmann law (the previous result gave the total energy density; here we're considering the actual rate of radiation) is therefore π times 0.6

$$j = \frac{2\pi^5 k_B^4}{15h^3c^2}T^4 = \frac{\pi^2 k_B^4}{60\hbar^3c^2}T^4 \tag{0.8}$$

[Note that this is just the total energy density from our earlier calculation multiplied by c.]

Example. If you want to become a star, let's see how brightly you shine. An average person's skin area is 1.4 m^2 and a normal skin temperature is about 306 K = 33°C (this is lower than normal body temperature of 37°C because your skin is on the outside!). Plugging in the numbers, we get

$$\begin{aligned} 1.4 j_r &= \frac{1.4\pi^2\left(1.3806488\times10^{-23}\right)^4(306)^4}{60\left(1.05457173\times10^{-34}\right)^3\left(2.99792458\times10^{8}\right)^2} && (0.9)\\ &= 696 \text{ watts} && (0.10)\end{aligned}$$

This is the outward radiation rate, but we would also be absorbing energy from the surroundings. If these surroundings are at a temperature of 293 K = 20°C, the power absorbed is

$$1.4 j_a = \left(\frac{293}{306}\right)^4 (1.4 j_r) = 585 \text{ watts} \tag{0.11}$$

so in a room temperature environment, we are radiating energy at a net rate of $696 - 585 = 111$ watts or about the rate of an old-fashioned lightbulb.

If we're emitting about the same energy as a lightbulb, how come we don't glow in the dark? The answer can be found from Wien's displacement law which gives the wavelength at which the maximum energy is emitted.

$$\lambda_{max} = \frac{2.901 \times 10^{-3}\text{m K}}{T} \tag{0.12}$$

$$= \frac{2.901 \times 10^{-3}\text{m K}}{306 \text{ K}} \tag{0.13}$$

$$= 9.46 \times 10^{-6} \text{ m} \tag{0.14}$$

This is in the infrared region of the spectrum so the radiation is invisible to the human eye, but is readily detectable by infrared sensors.

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