BLACKBODY RADIATION: CLASSICAL APPROXIMATIONS

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 3, Problems 3.10 - 3.12.

The blackbody radiation rate in terms of wavelength is

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)} \tag{1}$$

For very short wavelengths (high energies), $e^{hc/\lambda k_B T} \gg 1$ so we can approximate the rate by

$$B_\lambda \approx \frac{2hc^2}{\lambda^5} e^{-hc/\lambda k_B T} \tag{2}$$

This form matches the empirical formula obtained by Wien, which is

$$B_\lambda = \frac{a}{\lambda^5} e^{-b/T} \tag{3}$$

where a and b are constants determined by fitting the curve to experimental data.

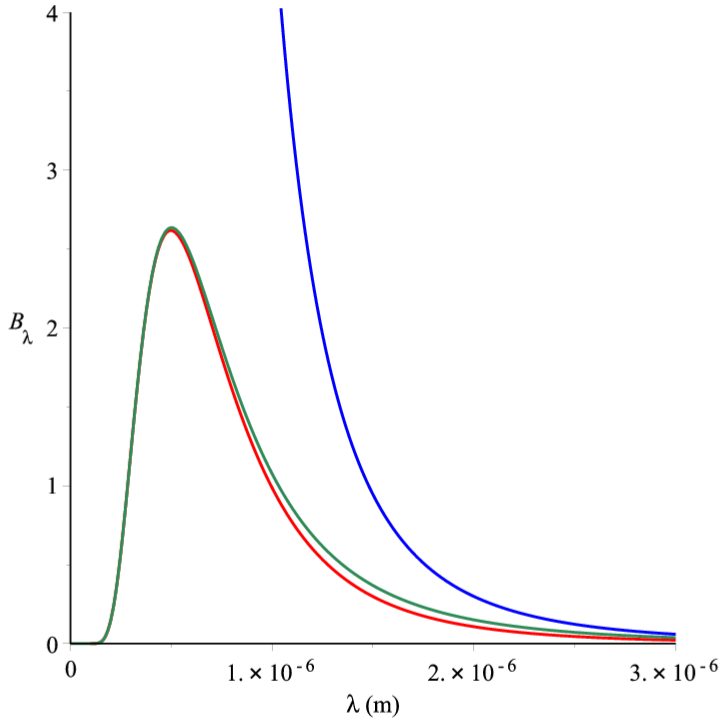
For long wavelengths, we can use the first order Taylor expansion for the exponential

$$e^{hc/\lambda k_B T} = 1 + \frac{hc}{\lambda k_B T} + \mathcal{O}\left(\frac{1}{\lambda^2}\right) \tag{4}$$

$$B_\lambda \approx \frac{2ck_B T}{\lambda^4} \tag{5}$$

This is the classical Rayleigh-Jeans law. Notice that Planck's constant has dropped out of the formula, and that if applied to all wavelengths it predicts that $B_\lambda \to \infty$ as $\lambda \to 0$. This is the *ultraviolet catastrophe* which was rectified with the introduction of the quantization of energy.

To compare the three curves for the Sun, where $T = 5777$ K, we have the following plot:



The green curve is Planck's formula, the blue curve is the Rayleigh-Jeans formula and the red curve is Wien's empirical formula. The Rayleigh-Jeans formula predicts twice the actual radiation at around $\lambda = 2000$ nm which is in the infrared region.

We've already derived Wien's displacement law for the wavelength at which the blackbody curve is maximum:

$$\lambda_{max} = \frac{2.901 \times 10^{-3}}{T} \text{ m} \tag{6}$$