

BLACKBODY RADIATION IN THE FREQUENCY DOMAIN

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 3, Problem 3.13.

The blackbody radiation rate in terms of wavelength is

$$(1) \quad B_{\lambda}(T) = \frac{2hc^2}{\lambda^5 (e^{hc/\lambda k_B T} - 1)}$$

B_{λ} is the rate at which a blackbody at temperature T radiates in watts per unit area, per unit wavelength band, per steradian. The amount of radiation emitted in a wavelength span $[\lambda, \lambda + d\lambda]$ is therefore $B_{\lambda} d\lambda$. To convert this to a rate per unit frequency interval, we need to convert $B_{\lambda} d\lambda$ to the equivalent form $B_{\nu} d\nu$. We have

$$(2) \quad \nu = \frac{c}{\lambda}$$

$$(3) \quad d\nu = -\frac{c}{\lambda^2} d\lambda$$

$$(4) \quad |B_{\lambda} d\lambda| = \left| \frac{\lambda^2}{c} B_{\lambda} d\nu \right|$$

$$(5) \quad B_{\nu} = \frac{\lambda^2}{c} B_{\lambda}$$

$$(6) \quad = \frac{2hc}{\lambda^3 (e^{hc/\lambda k_B T} - 1)}$$

$$(7) \quad = \frac{2h\nu^3}{c^2 (e^{h\nu/k_B T} - 1)}$$

The frequency at which B_{ν} is a maximum is found as usual by solving $dB_{\nu}/d\nu = 0$. As in the case of finding λ_{max} , this gives rise to an equation that must be solved numerically. We get

$$(8) \quad \frac{dB_\nu}{d\nu} = \frac{6h\nu^2}{c^2 (e^{h\nu/k_B T} - 1)} - \frac{2h^2\nu^3 e^{h\nu/k_B T}}{c^2 k_B T (e^{h\nu/k_B T} - 1)^2} = 0$$

$$(9) \quad 3 = e^{h\nu/k_B T} \left(3 - \frac{h\nu}{k_B T} \right)$$

$$(10) \quad 3 = e^x (3 - x)$$

where $x \equiv h\nu/k_B T$. Solving this using Maple we get $x = 2.821439372$ (or $x = 0$, but that isn't very interesting) and then plugging in $h = 6.62606957 \times 10^{-34}$ and $k_B = 1.3806488 \times 10^{-23}$ (in SI units) we get

$$(11) \quad \nu_{max} = 5.879 \times 10^{10} T \text{ s}^{-1}$$

For the Sun, $T = 5777$ K so $\nu_{max} = 3.396 \times 10^{14} \text{ s}^{-1}$ which corresponds to a wavelength of

$$(12) \quad \lambda = \frac{c}{\nu_{max}} = 880 \text{ nm}$$

This is in the infrared region. This is different from $\lambda_{max} = 2.901 \times 10^{-3}/T = 502 \text{ nm}$ as calculated from Wien's displacement law because B_λ and B_ν measure different things. B_λ is the radiation per unit wavelength interval and B_ν is the radiation per unit frequency interval and as we've seen from 3, these two intervals are not linearly related. The size of a frequency interval depends on the wavelength (and vice versa).