VEGA AS A BLACKBODY

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 3, Problem 3.16.

The blackbody radiation rate in a wavelength interval $d\lambda$ is

$$B_\lambda(T)\,d\lambda=\frac{2hc^2}{\lambda^5\left(e^{hc/\lambda k_BT}-1\right)}d\lambda \tag{0.1}$$

The luminosity L_λ within the same wavelength interval is the radiation rate times the surface area of the star:

$$L_\lambda d\lambda=4\pi R^2B_\lambda=\frac{8\pi hc^2R^2}{\lambda^5\left(e^{hc/\lambda k_BT}-1\right)}d\lambda \tag{0.2}$$

The flux F_λ received by an observer on Earth at a distance r from the star is then

$$F_\lambda d\lambda=\frac{L_\lambda}{4\pi r^2}=\frac{2hc^2}{\lambda^5\left(e^{hc/\lambda k_BT}-1\right)}\frac{R^2}{r^2}d\lambda \tag{0.3}$$

The apparent magnitude m in some wavelength interval (for example, one of the U, B and V magnitudes) is

$$m=M_{Sun}-2.5\log\frac{\int F_\lambda S_\lambda d\lambda}{\int F_{\lambda,10,Sun}S_\lambda d\lambda} \tag{0.4}$$

where M_{Sun} is the absolute magnitude of the Sun over the same wavelength interval and $F_{\lambda,10,Sun}$ is the flux F_λ for the Sun at a distance of 10 pc. The function S_λ is the *sensitivity function* and indicates what fraction of the star's light is received at a given wavelength λ. Thus $0\le S_\lambda\le1$ for all λ. Since the values for the Sun in 0.4 are same for every star, we can write it as

$$m = -2.5\log\int F_\lambda S_\lambda d\lambda + C \tag{0.5}$$

$$C \equiv M_{Sun} + 2.5\log\int F_{\lambda,10,Sun} S_\lambda d\lambda \tag{0.6}$$

C depends on the wavelength interval over which the integral is done, and on the sensitivity S_λ of the detector, so it will be different for different filters and telescopes. In practice, C_U, C_B and C_V are all chosen so that U, B and V are all zero for the star Vega. Unfortunately, this doesn't mean that Vega is actually the same brightness when viewed in these three regions of the spectrum. We can find the wavelength band in which Vega actually appears brightest by calculating the colour indices using the above formulas.

(0.7)

$$U - B = M_{Sun} - 2.5\log\left[\frac{\int F_\lambda S_\lambda d\lambda}{\int F_{\lambda,10,Sun} S_\lambda d\lambda}\right]_U - M_{Sun} + 2.5\log\left[\frac{\int F_\lambda S_\lambda d\lambda}{\int F_{\lambda,10,Sun} S_\lambda d\lambda}\right]_B$$

(0.8)

$$= 2.5\log\frac{\left[\int F_\lambda S_\lambda d\lambda\right]_B}{\left[\int F_\lambda S_\lambda d\lambda\right]_U}$$

(0.9)

$$= 2.5\log\frac{\left[\int B_\lambda S_\lambda d\lambda\right]_B}{\left[\int B_\lambda S_\lambda d\lambda\right]_U}$$

where in each case the integrals in square brackets are evaluated over the wavelength range corresponding to the subscript U or B. If the wavelength filters are narrow enough and we take the sensitivity function S_λ to be 1 inside the filter's range and 0 outside, we can approximate the integrals by calculating B_λ at the midpoint of the wavelength range and just multiplying by the filter's bandwidth $\Delta\lambda$. That is, we get

$$U - B \approx 2.5\log\frac{B_{\lambda_B}\Delta\lambda_B}{B_{\lambda_U}\Delta\lambda_U} \tag{0.10}$$

$$= 2.5\log\frac{\lambda_U^5\left(e^{hc/\lambda_U k_B T} - 1\right)\Delta\lambda_B}{\lambda_B^5\left(e^{hc/\lambda_B k_B T} - 1\right)\Delta\lambda_U} \tag{0.11}$$

There's a similar relation for $B - V$:

$$B-V \approx 2.5 \log \frac{\lambda_B^5 \left(e^{hc/\lambda_B k_B T}-1\right) \Delta\lambda_V}{\lambda_V^5 \left(e^{hc/\lambda_V k_B T}-1\right) \Delta\lambda_B} \tag{0.12}$$

The standard filters are

- U: 365 ± 34 nm;
- B: 440 ± 49 nm;
- V: 550 ± 44.5 nm.

Using a temperature of $T = 9600$ K for Vega, we get

$$U-B = +0.161 \tag{0.13}$$

$$B-V = -0.539 \tag{0.14}$$

Thus a blackbody with $T = 9600$ K as an approximation to Vega would appear brightest in the blue region, since $U > B$ and $B < V$ so B is the smallest (brightest) magnitude.

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