COLOUR INDICES IN TERMS OF BLACKBODY RADIATION RATE

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 3, Problem 3.18.

We can write the apparent magnitude m_λ of a star for some wavelength band in terms of the flux F_λ in that band received on Earth from the star as

$$m_\lambda = -2.5\log\int F_\lambda S_\lambda d\lambda + C_\lambda \tag{0.1}$$

The constantC_λ depends on the wavelength interval over which the integral is done, and on the sensitivity S_λ of the detector, so it will be different for different filters. If we measure the apparent magnitudes in the three standard bands U, B and V and treat the star as a blackbody, we can work out the three constants C_U, C_B and C_V. Since colour indices are commonly used to classify stars, we can work out similar equations for these indices.

$$U-B = -2.5\log\int F_U S_U d\lambda + C_U + 2.5\log\int F_B S_B d\lambda - C_B \tag{0.2}$$

$$= -2.5\log\frac{\int F_U S_U d\lambda}{\int F_B S_B d\lambda} + C_{U-B} \tag{0.3}$$

where $C_{U-B} \equiv C_U - C_B$. Since the argument of the log is now dimensionless, the constant C_{U-B} is independent of the units used to measure flux. In their example 3.6.2, Carroll & Ostlie use a star with surface temperature of $T = 42000$ K and measured colour indices of $U-B = -1.19$ and $B-V = -0.33$. The standard filters are

- U: 365 ± 34 nm;
- B: 440 ± 49 nm;
- V: 550 ± 44.5 nm.

If we make the assumptions that $S = 1$ for each filter within these bands and $S = 0$ outside these bands, and that the flux doesn't change much over the bandwidth in each filter, we can approximate the relation above by

$$U-B\approx-2.5\log\frac{F_U\Delta\lambda_U}{F_B\Delta\lambda_B}+C_{U-B} \tag{0.4}$$

with a similar relation for $B-V$:

$$B-V\approx-2.5\log\frac{F_B\Delta\lambda_B}{F_V\Delta\lambda_V}+C_{B-V} \tag{0.5}$$

As we're dealing with ratios of flux for the same star, we can express these equations in terms of the blackbody radiation rate:

$$\begin{aligned}
U-B &= -2.5\log\frac{B_U\Delta\lambda_U}{B_B\Delta\lambda_B}+C_{U-B} && (0.6)\\
&= -2.5\log\frac{\lambda_B^5\left(e^{hc/\lambda_B k_B T}-1\right)\Delta\lambda_U}{\lambda_U^5\left(e^{hc/\lambda_U k_B T}-1\right)\Delta\lambda_B}+C_{U-B} && (0.7)\\
B-V &= -2.5\log\frac{\lambda_V^5\left(e^{hc/\lambda_V k_B T}-1\right)\Delta\lambda_B}{\lambda_B^5\left(e^{hc/\lambda_B k_B T}-1\right)\Delta\lambda_V}+C_{B-V} && (0.8)
\end{aligned}$$

Plugging in the values for the star mentioned above, we get

$$\begin{aligned}
C_{U-B} &= -0.87 && (0.9)\\
C_{B-V} &= +0.65 && (0.10)
\end{aligned}$$

Given these values, we can now estimate the colour indices for any star if we know its temperature. For the Sun, $T=5777$ K and plugging this into 0.7 and 0.8, we get

$$\begin{aligned}
U-B &= -0.222 && (0.11)\\
B-V &= +0.571 && (0.12)
\end{aligned}$$

The measured values given by Carroll & Ostlie are $U-B=+0.195$ and $B-V=+0.65$ so the agreement isn't great, especially for $U-B$.