LENGTH CONTRACTION AND TIME DILATION: A FEW EXAMPLES

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 4, Problems 4.4-4.5.

Here are a few examples of length contraction and time dilation.

Example 1. A rod is moving at a speed β relative to an observer S such that S measures the length of the rod to be half its rest length. The speed can be found from

$$L = \frac{L_0}{\gamma} = \frac{L_0}{2} \tag{0.1}$$

$$\gamma = 2 = \frac{1}{\sqrt{1-\beta^2}} \tag{0.2}$$

$$\beta = \sqrt{1-1/\gamma^2} \tag{0.3}$$

$$= \frac{\sqrt{3}}{2} = 0.866 \tag{0.4}$$

Example 2. A train travelling at speed $\beta = 0.8$ passes an observer S standing on a platform whose rest length is $P_0 = 60$ m. In frame S, the ends of the train are observed to be exactly at the ends of the platform at the same time $t = 0$. In other words, S measures the length of the train to be 60 m.

S measures the time taken for the train to pass him as

$$t = \frac{60}{0.8} = 75 \text{ m} \tag{0.5}$$

[All times are divided by c but we're using units such that $c = 1$ so that time is measured in metres.]

S measures the length of the train to be contracted from its rest length by the factor γ, so an observer S' on the train measures the train's length as

$$L_0 = 60\gamma \tag{0.6}$$
$$= \frac{60}{\sqrt{1-(0.8)^2}} \tag{0.7}$$
$$= \frac{5}{3}60 = 100 \text{ m} \tag{0.8}$$

The rest length of the platform is $P_0 = 60$ m so to S' the platform appears contracted to a length

$$P = \frac{P_0}{\gamma} = \frac{3}{5}60 = 36 \text{ m} \tag{0.9}$$

To find how long S' thinks it takes for the train to pass S, we need to realize that S' must use *two* clocks in his frame to do this measurement (one at each end of the train), as opposed to the single clock that S uses to measure how long it takes the train to pass him. Therefore, it is the S frame measurement that is slow, and the time measured by S' is

$$\Delta t' = \gamma\Delta t \tag{0.10}$$
$$= \frac{5}{3}75 = 125 \text{ m} \tag{0.11}$$

Although S says that the two ends of the train are at the two ends of the platform at the same time $t = 0$, S' will not observe the two ends of the train to be at the ends of the platform at the same time. Suppose we define the origins of the two frames to coincide when the rear of the train is at the rear end of the platform, so that

$$(t_r, x_r) = \left(t'_r, x'_r\right) = (0,0) \tag{0.12}$$

In frame S, the front end of the train is at the front of the platform at $t_f = 0$, so

$$\left(t_f, x_f\right) = (0,60) \tag{0.13}$$

We can use a Lorentz transformation to find t'_f:

$$t'_f = \gamma\left(t_f - \beta x_f\right) \tag{0.14}$$
$$= -60\gamma\beta \tag{0.15}$$
$$= -80 \text{ m} \tag{0.16}$$

So S' thinks the front end passed the front of the platform 80 m before the back end passes the back of the platform.

To check this is consistent with the results above, S' thinks the platform is 36 m long so it will take the back end of the train a time of $36/0.8 = 45$ m to travel the length of the platform, so that it passes the front end at $t' = +45$ m. The total time taken for the train to pass the front end of the platform is thus $t = 80 + 45 = 125$ m, in agreement with the time taken to pass S calculated above.