

RELATIVISTIC ACCELERATION IN TERMS OF FORCE

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 4, Problem 4.14.

Ordinary force in relativity is given by

$$\mathbf{F} = \frac{m}{\sqrt{1-u^2/c^2}} \left[\mathbf{a} + \frac{(\mathbf{u} \cdot \mathbf{a}) \mathbf{u}}{c^2 - u^2} \right] \quad (1)$$

To get a general expression for the acceleration \mathbf{a} in terms of the force, we take the dot product of both sides with the velocity \mathbf{u} :

$$\mathbf{u} \cdot \mathbf{F} = \gamma m (\mathbf{u} \cdot \mathbf{a}) \left(1 + \frac{u^2}{c^2 - u^2} \right) \quad (2)$$

$$= \frac{\gamma m c^2 (\mathbf{u} \cdot \mathbf{a})}{c^2 - u^2} \quad (3)$$

$$= \gamma^3 m (\mathbf{u} \cdot \mathbf{a}) \quad (4)$$

$$\mathbf{u} \cdot \mathbf{a} = \frac{\mathbf{u} \cdot \mathbf{F}}{\gamma^3 m} \quad (5)$$

Substituting back into 1, we get

$$\mathbf{a} = \frac{\mathbf{F}}{\gamma m} - \frac{\mathbf{u}}{c^2 - u^2} \frac{\mathbf{u} \cdot \mathbf{F}}{\gamma^3 m} \quad (6)$$

$$= \frac{\mathbf{F}}{\gamma m} - \frac{\mathbf{u}}{\gamma m c^2} (\mathbf{u} \cdot \mathbf{F}) \quad (7)$$

In the limit of small \mathbf{u} , this reduces to the familiar Newton's law $\mathbf{F} = m\mathbf{a}$, but in the relativistic region, the acceleration depends on the object's velocity. As a result, the acceleration isn't parallel to the force unless \mathbf{F} is either parallel to \mathbf{u} or $\mathbf{F} \perp \mathbf{u}$; in the latter case $\mathbf{u} \cdot \mathbf{F} = 0$ and the second term is zero.

PINGBACKS

Pingback: Acceleration under a constant force