RELATIVISTIC ACCELERATION IN TERMS OF FORCE

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 4, Problem 4.14.

Ordinary force in relativity is given by

$$\mathbf{F} = \frac{m}{\sqrt{1-u^2/c^2}}\left[\mathbf{a}+\frac{(\mathbf{u}\cdot\mathbf{a})\,\mathbf{u}}{c^2-u^2}\right] \tag{1}$$

To get a general expression for the acceleration $\mathbf{a}$ in terms of the force, we take the dot product of both sides with the velocity $\mathbf{u}$:

$$\begin{aligned}
\mathbf{u}\cdot\mathbf{F} &= \gamma m\,(\mathbf{u}\cdot\mathbf{a})\left(1+\frac{u^2}{c^2-u^2}\right) &(2)\\
&= \frac{\gamma mc^2\,(\mathbf{u}\cdot\mathbf{a})}{c^2-u^2} &(3)\\
&= \gamma^3 m\,(\mathbf{u}\cdot\mathbf{a}) &(4)\\
\mathbf{u}\cdot\mathbf{a} &= \frac{\mathbf{u}\cdot\mathbf{F}}{\gamma^3 m} &(5)
\end{aligned}$$

Substituting back into 1, we get

$$\begin{aligned}
\mathbf{a} &= \frac{\mathbf{F}}{\gamma m}-\frac{\mathbf{u}}{c^2-u^2}\frac{\mathbf{u}\cdot\mathbf{F}}{\gamma^3 m} &(6)\\
&= \frac{\mathbf{F}}{\gamma m}-\frac{\mathbf{u}}{\gamma mc^2}\,(\mathbf{u}\cdot\mathbf{F}) &(7)
\end{aligned}$$

In the limit of small $\mathbf{u}$, this reduces to the familiar Newton's law $\mathbf{F} = m\mathbf{a}$, but in the relativistic region, the acceleration depends on the object's velocity. As a result, the acceleration isn't parallel to the force unless $\mathbf{F}$ is either parallel to $\mathbf{u}$ or $\mathbf{F}\perp\mathbf{u}$; in the latter case $\mathbf{u}\cdot\mathbf{F}=0$ and the second term is zero.

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