ACCELERATION UNDER A CONSTANT FORCE

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 4, Problem 4.15.

Under a constant force F, an object undergoes hyperbolic motion. In one dimension for a constant force we have

$$\frac{dp}{dt} = F \tag{1}$$

$$p = Ft + C \tag{2}$$

where C is a constant of integration. If the object starts at $t=0$ at rest (in the lab frame), then $C=0$, and

$$p = \frac{mu}{\sqrt{1-u^2/c^2}} = Ft \tag{3}$$

which can be solved for the velocity u to give

$$u = \frac{F}{m}\frac{t}{\sqrt{1+(Ft/mc)^2}} \tag{4}$$

We can also get this formula by integrating the expression for the acceleration

$$\mathbf{a} = \frac{\mathbf{F}}{\gamma m} - \frac{\mathbf{u}}{\gamma mc^2}(\mathbf{u}\cdot\mathbf{F}) \tag{5}$$

If $\mathbf{F}$ is constant and acts on an object initially at rest, then $\mathbf{F} \parallel \mathbf{u}$ and

$$a = \frac{du}{dt} \tag{6}$$

$$= \frac{F}{\gamma m}\left(1-\frac{u^2}{c^2}\right) \tag{7}$$

$$= \frac{F}{m}\left(1-\frac{u^2}{c^2}\right)^{3/2} \tag{8}$$

To find $u(t)$ we integrate:

$$\int\left(1-\frac{u^2}{c^2}\right)^{-3/2} du = \frac{F}{m}\int dt \tag{9}$$

$$\frac{u}{\sqrt{1-u^2/c^2}} = \frac{F}{m}(t+t_0) \tag{10}$$

If $u=0$ at $t=0$, the constant of integration is $t_0=0$ and we get 3 again. From 3 we can get the inverse function

$$t = \frac{mu}{F\sqrt{1-u^2/c^2}} \tag{11}$$

Thus as $u \to c$, $t \to \infty$ so the object never quite reaches the speed of light.

Because of the velocity-dependent term in 5, the acceleration due to a constant force is not constant, but rather decreases as u increases. If we start with $a_0 = F/m = g = 9.8\text{ m s}^{-2}$, then the times required to reach various speeds are found from

$$t = \frac{u}{9.8\sqrt{1-u^2/c^2}} \tag{12}$$

$\frac{u}{c}$	t
0.9	6.3×10^{7} s $=2$ years
0.99	2.14×10^{8} s $=6.78$ years
0.999	6.82×10^{8} s $=21.6$ years
0.9999	2.16×10^{9} s $=68.4$ years
1.0	∞

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