

ACCELERATION UNDER A CONSTANT FORCE

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 4, Problem 4.15.

Under a constant force F , an object undergoes hyperbolic motion. In one dimension for a constant force we have

$$\frac{dp}{dt} = F \quad (1)$$

$$p = Ft + C \quad (2)$$

where C is a constant of integration. If the object starts at $t = 0$ at rest (in the lab frame), then $C = 0$, and

$$p = \frac{mu}{\sqrt{1 - u^2/c^2}} = Ft \quad (3)$$

which can be solved for the velocity u to give

$$u = \frac{F}{m} \frac{t}{\sqrt{1 + (Ft/mc)^2}} \quad (4)$$

We can also get this formula by integrating the expression for the acceleration

$$\mathbf{a} = \frac{\mathbf{F}}{\gamma m} - \frac{\mathbf{u}}{\gamma mc^2} (\mathbf{u} \cdot \mathbf{F}) \quad (5)$$

If \mathbf{F} is constant and acts on an object initially at rest, then $\mathbf{F} \parallel \mathbf{u}$ and

$$a = \frac{du}{dt} \quad (6)$$

$$= \frac{F}{\gamma m} \left(1 - \frac{u^2}{c^2}\right) \quad (7)$$

$$= \frac{F}{m} \left(1 - \frac{u^2}{c^2}\right)^{3/2} \quad (8)$$

To find $u(t)$ we integrate:

$$\int \left(1 - \frac{u^2}{c^2}\right)^{-3/2} du = \frac{F}{m} \int dt \quad (9)$$

$$\frac{u}{\sqrt{1 - u^2/c^2}} = \frac{F}{m} (t + t_0) \quad (10)$$

If $u = 0$ at $t = 0$, the constant of integration is $t_0 = 0$ and we get 3 again. From 3 we can get the inverse function

$$t = \frac{mu}{F\sqrt{1 - u^2/c^2}} \quad (11)$$

Thus as $u \rightarrow c$, $t \rightarrow \infty$ so the object never quite reaches the speed of light.

Because of the velocity-dependent term in 5, the acceleration due to a constant force is not constant, but rather decreases as u increases. If we start with $a_0 = F/m = g = 9.8 \text{ m s}^{-2}$, then the times required to reach various speeds are found from

$$t = \frac{u}{9.8\sqrt{1 - u^2/c^2}} \quad (12)$$

$\frac{u}{c}$	t
0.9	$6.3 \times 10^7 \text{ s} = 2 \text{ years}$
0.99	$2.14 \times 10^8 \text{ s} = 6.78 \text{ years}$
0.999	$6.82 \times 10^8 \text{ s} = 21.6 \text{ years}$
0.9999	$2.16 \times 10^9 \text{ s} = 68.4 \text{ years}$
1.0	∞

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