# BARNARD'S STAR: DISTANCE AND VELOCITY

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problem 5.1.

By combining measurements of parallax, proper motion (the apparent motion of a star across the sky, usually measured in seconds of arc per year) and the Doppler shifting of spectral lines, we can work out a star's actual velocity through space. A good example is Barnard's star, a magnitude 9.5 red dwarf star in the constellation Ophiucus, which has the largest known proper motion of any star.

The parallax is $p=0.54901''$ so its distance is

$$d=\frac{1}{p}=1.82146\text{ pc}=5.9408\text{ ly}=5.62032\times10^{16}\text{ m} \tag{0.1}$$

Its proper motion is $\mu=10.3577''\text{ yr}^{-1}$ which works out to

$$\mu = 10.3577\frac{\pi}{3600\times180}=5.021555\times10^{-5}\text{ rad yr}^{-1} \tag{0.2}$$

$$= \frac{5.021555\times10^{-5}}{3600\times24\times365.25}=1.59123\times10^{-12}\text{ rad s}^{-1} \tag{0.3}$$

At the distance of Barnard's star, this gives a transverse velocity component of

$$v_\theta=d\mu=8.9432\times10^{4}\text{ m s}^{-1} \tag{0.4}$$

In the spectrum, the hydrogen alpha (H$\alpha$) line is observed at a wavelength of 656.034 nm, and the rest H$\alpha$ wavelength is 656.28 nm. As the star's wavelength is shorter than the rest wavelength, it is blue-shifted and Barnard's star is moving towards us. The (relativistic) Doppler shift formula for a speed $v$ that is positive for approaching objects is

$$\bar{\lambda}=\lambda\sqrt{\frac{1-v/c}{1+v/c}} \tag{0.5}$$

For non-relativistic speeds, $v \ll c$ and we can Taylor-expand this formula to get

$$\bar{\lambda} = \lambda - \frac{\lambda}{c} v + \mathscr{O}\left(\frac{v^2}{c^2}\right) \tag{0.6}$$

Keeping only terms first order in $v/c$ we get

$$v = \frac{\lambda - \bar{\lambda}}{\lambda} c \tag{0.7}$$

For Barnard's star this gives a radial velocity of

$$\begin{aligned} v_r &= \frac{656.28 - 656.034}{656.28}\left(2.99792 \times 10^8\right) && (0.8) \\ &= 1.12374 \times 10^5 \text{ m s}^{-1} && (0.9) \end{aligned}$$

The actual speed of Barnard's star through space, relative to Earth, is

$$v = \sqrt{v_r^2 + v_\theta^2} = 1.4362 \times 10^5 \text{ m s}^{-1} \tag{0.10}$$