

## DIFFRACTION GRATINGS IN SPECTROSCOPY

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problem 5.2.

Jenkins, Francis A. & White, Harvey E. (1957), *Fundamentals of Optics*, 3rd Edition; McGraw-Hill - Chapter 17.

One of the main sources of information about celestial objects is their spectrum. In the case of light from a star, the spectrum can tell us a lot about the composition of the star, and the Doppler shift can tell us the star's radial velocity.

Getting a detailed spectrum of a star is somewhat more involved than just passing its light through a prism, as Newton famously did to show that white light is composed of the colours of the rainbow. In practice, a *diffraction grating* is used to split starlight into its constituent spectral lines. A complete treatment of the physics of diffraction gratings is beyond the scope of this post, so if you're interested I'll refer you to any good textbook on optics. [My own reference, the book by Jenkins and White mentioned at the top, is quite dated now, but I doubt that the physics has changed in the intervening years.] I'll give a summary of the key concepts here.

A diffraction grating is basically a screen with a large number of closely-spaced parallel slits cut in it (actual gratings are usually made of glass with very fine lines ruled on them, but the principle is the same). Consider a grating with only 2 slits separated by a distance  $d$ . Monochromatic light of wavelength  $\lambda$  is shone onto the grating and diffracts through each of the slits. [Diffraction is the process by which a light wave spreads out after passing through a narrow gap. The same effect can be seen with water waves as they hit a narrow gap in a barrier; waves spread out in a semi-circular pattern beyond the gap.]

Now look at the light rays that leave the slits at an angle  $\theta$  to the normal to the plane containing the slits. By drawing a diagram (see Fig 3.3 in Carroll & Ostlie, although in practice the two light rays leaving the slits are parallel and are focussed onto the detecting screen by a convex lens or concave mirror) we can see that the path difference between the two rays is  $d \sin \theta$ . If this path difference is an integral multiple of the wavelength, the two rays will reinforce each other and we'll see a bright fringe at that angle. On the other hand, if the path difference is an integral multiple of wavelengths

plus half a wavelength, the two rays will destructively interfere, cancelling each other out, and we'll see a dark fringe. That is, the condition for bright fringes is

$$(1) \quad d \sin \theta = n\lambda$$

for  $n$  a non-negative integer.

Now suppose we have a diffraction grating with  $N$  slits. The same condition applies for light fringes, but because we're now adding up  $N$  rays instead of just 2, the intensity of the bright fringes is larger, and is in fact proportional to  $N^2$ . The actual formula for the intensity is

$$(2) \quad I = a^2 \frac{\sin^2 N\gamma}{\sin^2 \gamma}$$

where  $a$  is a constant, derived from the intensity passing through a single slit, and

$$(3) \quad \gamma = \frac{\pi d \sin \theta}{\lambda}$$

The bright fringes occur when  $\gamma = n\pi$ . This can be seen by finding the limit:

$$(4) \quad \lim_{\gamma \rightarrow n\pi} \frac{\sin N\gamma}{\sin \gamma}$$

This limit can be found using l'Hôpital's rule from calculus, which states that for two functions  $f$  and  $g$  where  $f(x_0) = g(x_0) = 0$ :

$$(5) \quad \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow x_0} \frac{f'(x)}{g'(x)}$$

Therefore

$$(6) \quad \lim_{\gamma \rightarrow n\pi} \frac{\sin N\gamma}{\sin \gamma} = \lim_{\gamma \rightarrow n\pi} \frac{N \cos N\gamma}{\cos \gamma} = \pm N$$

$$(7) \quad \lim_{\gamma \rightarrow n\pi} \frac{\sin^2 N\gamma}{\sin^2 \gamma} = N^2$$

Because of the  $N$  in the upper sine, however, the numerator has many more zeroes than the denominator, meaning that whenever  $N\gamma = m\pi$  but  $m$  is not a multiple of  $N$ ,  $I = 0$ . Between any two bright fringes there will

therefore be  $N - 1$  dark fringes and  $N - 2$  secondary 'brightish' fringes between these dark fringes. In practice, these secondary fringes are much fainter than the primary bright fringes and for large  $N$  they are effectively invisible.

It turns out that the number of slits  $N$  also determines the angular width of each primary maximum, according to

$$(8) \quad \Delta\theta_{width} = \frac{2\lambda}{Nd \cos \theta}$$

That is, the more slits, the sharper the spectral line for a given wavelength. Looked at another way, the smallest wavelength difference  $\Delta\lambda$  that can be resolved is

$$(9) \quad \Delta\lambda = \frac{\lambda}{nN}$$

Not only does increasing the number slits increase the resolving power of the grating, but looking a higher order (larger  $n$ ) lines allows greater resolution.

**Example.** The sodium D lines are commonly found in stellar spectra and have wavelengths of  $\lambda_1 = 588.997$  nm and  $\lambda_2 = 589.594$  nm. If light containing these two lines shines on a grating with 300 lines per millimetre, then

$$(10) \quad d = \frac{10^{-3}}{300} = 3.33 \times 10^{-6} \text{ m}$$

From 1 with  $n = 2$  (second-order spectra) we have

$$(11) \quad \sin \theta_1 = \frac{2(588.997 \times 10^{-9})}{3.33 \times 10^{-6}} = 0.3533982$$

$$(12) \quad \theta_1 = 20.69531^\circ$$

$$(13) \quad \sin \theta_2 = \frac{2(589.594 \times 10^{-9})}{3.33 \times 10^{-6}} = 0.3537564$$

$$(14) \quad \theta_2 = 20.71725^\circ$$

The angle between the two lines is therefore

$$(15) \quad \Delta\theta_{Na} = \theta_2 - \theta_1 = 0.02194^\circ$$

From 9 we can work out how many lines need to be illuminated in order for these 2 lines to be resolved in second-order. For the wavelength in the formula, we'll use the average of the two wavelengths of the sodium D lines.

$$\begin{aligned} (16) \quad N &= \frac{\lambda}{n\Delta\lambda} \\ (17) \quad &= \frac{589.2955 \times 10^{-9}}{2 \times 0.597 \times 10^{-9}} \\ (18) \quad &= 494 \end{aligned}$$

Thus if only 2 mm of the grating were illuminated, we could resolve the lines, at least at second-order.

#### PINGBACKS

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Pingback: [Radio interferometry](#)