COMPTON EFFECT; COMPTON WAVELENGTH

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problems 5.5 - 5.7.

A while back, we derived the formula for the Compton effect, which is the change in the frequency of a photon when it scatters off a stationary electron. The formula, in relativistic units ($c=1$) is

$$\frac{1}{\nu'}=\frac{1}{\nu}+\frac{h}{m}(1-\cos\theta) \tag{0.1}$$

To convert this formula to wavelengths and to restore c, we first reinsert c into the above equation. The units of ν are s^{-1}, and of m are kg^{-1}. The units of Planck's constant can be found from its definition in the formula

$$E=h\nu \tag{0.2}$$

Thus the units of h are $(\text{energy})\times(\text{time})=\text{kg m}^2\text{s}^{-1}$. This also happens to be the units of angular momentum, since $\mathbf{L}=\mathbf{r}\times\mathbf{p}$, so the units of $\mathbf{L}$ are $\text{m}\times\text{kg m s}^{-1}=\text{kg m}^2\text{s}^{-1}$. From 0.1, the units of the last term on the RHS must be seconds, so with c present explicitly, we must have

$$\frac{1}{\nu'}=\frac{1}{\nu}+\frac{h}{mc^2}(1-\cos\theta) \tag{0.3}$$

In terms of wavelength, we have

$$\lambda\nu=c \tag{0.4}$$

so

$$\frac{\lambda'}{c}=\frac{\lambda}{c}+\frac{h}{mc^2}(1-\cos\theta) \tag{0.5}$$

$$\Delta\lambda=\frac{h}{mc}(1-\cos\theta) \tag{0.6}$$

The change in wavelength is inversely proportional to the mass of the stationary object off which the photon scatters. For an electron

(0.7)
$$\frac{h}{m_e c} = \frac{6.62606957 \times 10^{-34}}{(9.10938291 \times 10^{-31})(2.99792458 \times 10^{8})} = 2.426 \times 10^{-12} \text{ m}$$

This is the *Compton wavelength* of the electron. The wavelength shift is independent of the wavelength of the incoming photon, so for visible light, where λ is around 5×10^{-7} m, the Compton effect is negligible. It becomes noticeable only for much shorter wavelengths (and thus much higher energy photons). For X-rays and gamma rays, λ is in the range $10^{-12} - 10^{-11}$ m so the Compton shift is comparable with the original wavelength.

For a heavier particle, such as a proton, the Compton effect is correspondingly smaller. The Compton wavelength of a proton is

(0.8)
$$\frac{h}{m_p c} = \frac{6.62606957 \times 10^{-34}}{(1.67262178 \times 10^{-27})(2.99792458 \times 10^{8})} = 1.321 \times 10^{-15} \text{ m}$$