

## GRAVITATIONAL HYDROGEN ATOM

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problem 5.9.

To see that gravity is much weaker than the electrostatic force, we can repeat Bohr's semi-classical derivation of the hydrogen energy levels, replacing the Coulomb force with the Newtonian gravitational force. We can do this with the following replacement:

$$(1) \quad \frac{e^2}{4\pi\epsilon_0} \rightarrow Gm_em_p$$

giving energy levels of

$$(2) \quad E = - (Gm_em_p)^2 \frac{m_e}{2n^2\hbar^2}$$

$$(3) \quad = - \frac{4.23 \times 10^{-97} \text{ J}}{n^2}$$

$$(4) \quad = - \frac{2.64 \times 10^{-78} \text{ eV}}{n^2}$$

compared to the actual energy levels of hydrogen:

$$(5) \quad E = - \frac{13.6 \text{ eV}}{n^2}$$

The radii are

$$(6) \quad r_n = \frac{n^2\hbar^2}{Gm_e^2m_p}$$

$$(7) \quad = (1.2 \times 10^{29} \text{ m}) n^2$$

$$(8) \quad = (1.2 \times 10^{38} \text{ nm}) n^2$$

$$(9) \quad = (8 \times 10^{17} \text{ AU}) n^2$$

$$(10) \quad = (1.27 \times 10^{13} \text{ ly}) n^2$$

Thus the ground state radius of gravitational hydrogen is many times larger than the visible universe, compared with the electrostatic Bohr radius of  $5.29177 \times 10^{-11}$  m.