GRAVITATIONAL HYDROGEN ATOM

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problem 5.9.

To see that gravity is much weaker than the electrostatic force, we can repeat Bohr's semi-classical derivation of the hydrogen energy levels, replacing the Coulomb force with the Newtonian gravitational force. We can do this with the following replacement:

$$\frac{e^2}{4\pi\epsilon_0} \to Gm_e m_p \tag{1}$$

giving energy levels of

$$\begin{aligned} E &= -\left(Gm_e m_p\right)^2 \frac{m_e}{2n^2\hbar^2} && (2)\\ &= -\frac{4.23\times10^{-97}\text{ J}}{n^2} && (3)\\ &= -\frac{2.64\times10^{-78}\text{ eV}}{n^2} && (4) \end{aligned}$$

compared to the actual energy levels of hydrogen:

$$E = -\frac{13.6\text{ eV}}{n^2} \tag{5}$$

The radii are

$$\begin{aligned} r_n &= \frac{n^2\hbar^2}{Gm_e^2 m_p} && (6)\\ &= \left(1.2\times10^{29}\text{ m}\right)n^2 && (7)\\ &= \left(1.2\times10^{38}\text{ nm}\right)n^2 && (8)\\ &= \left(8\times10^{17}\text{ AU}\right)n^2 && (9)\\ &= \left(1.27\times10^{13}\text{ ly}\right)n^2 && (10) \end{aligned}$$

Thus the ground state radius of gravitational hydrogen is many times larger than the visible universe, compared with the electrostatic Bohr radius of 5.29177×10^{-11} m.