

HYDROGEN SPECTRUM: LYMAN, BALMER AND PASCHEN SERIES

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problems 5.10 - 5.11.

The energy levels in the hydrogen atom are

$$(1) \quad E_n = - \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{\mu}{2n^2\hbar^2} = - \frac{13.6 \text{ eV}}{n^2}$$

When the electron jumps between these levels it absorbs (when jumping to a higher level) or emits (lower) a photon with an energy $h\nu$ equal to the difference between the energy levels. This gives rise to a characteristic spectrum which can be observed in stars.

For example, when an electron makes the transition from $n = 3$ to $n = 1$, it can do so directly, or via the pair of transitions $3 \rightarrow 2 \rightarrow 1$. In the former case

$$(2) \quad E_{3 \rightarrow 1} = 13.6 \text{ eV} \left(1 - \frac{1}{3^2} \right) = 12.09 \text{ eV}$$

which corresponds to a wavelength of

$$(3) \quad \lambda_{3 \rightarrow 1} = \frac{c}{\nu} = \frac{hc}{E} = \frac{1240 \text{ eV nm}}{12.09 \text{ eV}} = 102.6 \text{ nm}$$

This is in the ultraviolet.

The pair of transitions gives

$$(4) \quad E_{3 \rightarrow 2} = 13.6 \text{ eV} \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 1.89 \text{ eV}$$

$$(5) \quad \lambda_{3 \rightarrow 2} = \frac{1240}{1.89} = 656.47 \text{ nm (visible)}$$

$$(6) \quad E_{2 \rightarrow 1} = 13.6 \text{ eV} \left(1 - \frac{1}{2^2} \right) = 10.2 \text{ eV}$$

$$(7) \quad \lambda_{2 \rightarrow 1} = \frac{1240}{10.2} = 121.6 \text{ nm (ultraviolet)}$$

Transitions to and from $n = 1$ give the Lyman series (all in the ultraviolet); to and from $n = 2$ (from higher energy levels) give the Balmer series (visible) and to and from $n = 3$ (from higher energy levels) give the Paschen series (infrared). For all series, the most energetic photons come from transitions from essentially infinite n to the base level. For Lyman, for example,

$$(8) \quad E_{n \rightarrow 1} = 13.6 \text{ eV} \left(1 - \frac{1}{n^2} \right)$$

so the *series limit* occurs as $n \rightarrow \infty$ and is 13.6 eV. This corresponds to a wavelength of

$$(9) \quad \lambda_1 = \frac{1240}{13.6} = 91.18 \text{ nm (ultraviolet)}$$

For Balmer, the series limit is

$$(10) \quad E_{n \rightarrow 2} = 13.6 \text{ eV} \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \rightarrow 3.4 \text{ eV}$$

$$(11) \quad \lambda_2 = \frac{1240}{3.4} = 364.7 \text{ nm (slightly ultraviolet)}$$

For Paschen,

$$(12) \quad E_{n \rightarrow 3} = 13.6 \text{ eV} \left(\frac{1}{3^2} - \frac{1}{n^2} \right) \rightarrow 1.51 \text{ eV}$$

$$(13) \quad \lambda_3 = \frac{1240}{1.51} = 820.6 \text{ nm (slightly infrared)}$$

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