HYDROGEN SPECTRUM: LYMAN, BALMER AND PASCHEN SERIES

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problems 5.10 - 5.11.

The energy levels in the hydrogen atom are

$$E_n = -\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{\mu}{2n^2\hbar^2} = -\frac{13.6 \text{ eV}}{n^2} \tag{1}$$

When the electron jumps between these levels it absorbs (when jumping to a higher level) or emits (lower) a photon with an energy $h\nu$ equal to the difference between the energy levels. This gives rise to a characteristic spectrum which can be observed in stars.

For example, when an electron makes the transition from $n=3$ to $n=1$, it can do so directly, or via the pair of transitions $3 \to 2 \to 1$. In the former case

$$E_{3\to 1} = 13.6 \text{ eV}\left(1 - \frac{1}{3^2}\right) = 12.09 \text{ eV} \tag{2}$$

which corresponds to a wavelength of

$$\lambda_{3\to 1} = \frac{c}{\nu} = \frac{hc}{E} = \frac{1240 \text{ eV nm}}{12.09 \text{ eV}} = 102.6 \text{ nm} \tag{3}$$

This is in the ultraviolet.

The pair of transitions gives

$$\begin{aligned}
E_{3\to 2} &= 13.6 \text{ eV}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89 \text{ eV} && (4)\\
\lambda_{3\to 2} &= \frac{1240}{1.89} = 656.47 \text{ nm (visible)} && (5)\\
E_{2\to 1} &= 13.6 \text{ eV}\left(1 - \frac{1}{2^2}\right) = 10.2 \text{ eV} && (6)\\
\lambda_{2\to 1} &= \frac{1240}{10.2} = 121.6 \text{ nm (ultraviolet)} && (7)
\end{aligned}$$

Transitions to and from $n=1$ give the Lyman series (all in the ultraviolet); to and from $n=2$ (from higher energy levels) give the Balmer series (visible) and to and from $n=3$ (from higher energy levels) give the Paschen series (infrared). For all series, the most energetic photons come from transitions from essentially infinite n to the base level. For Lyman, for example,

$$E_{n\to1}=13.6\text{ eV}\left(1-\frac{1}{n^{2}}\right) \tag{8}$$

so the *series limit* occurs as $n\to\infty$ and is 13.6 eV. This corresponds to a wavelength of

$$\lambda_{1}=\frac{1240}{13.6}=91.18\text{ nm (ultraviolet)} \tag{9}$$

For Balmer, the series limit is

$$E_{n\to2} = 13.6\text{ eV}\left(\frac{1}{2^{2}}-\frac{1}{n^{2}}\right)\to3.4\text{ eV} \tag{10}$$

$$\lambda_{2} = \frac{1240}{3.4}=364.7\text{ nm (slightly ultraviolet)} \tag{11}$$

For Paschen,

$$E_{n\to3} = 13.6\text{ eV}\left(\frac{1}{3^{2}}-\frac{1}{n^{2}}\right)\to1.51\text{ eV} \tag{12}$$

$$\lambda_{3} = \frac{1240}{1.51}=820.6\text{ nm (slightly infrared)} \tag{13}$$

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