

## HYDROGEN SPECTRUM: LYMAN, BALMER AND PASCHEN SERIES

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problems 5.10 - 5.11.

The energy levels in the hydrogen atom are

$$E_n = - \left( \frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{\mu}{2n^2\hbar^2} = - \frac{13.6 \text{ eV}}{n^2} \quad (1)$$

When the electron jumps between these levels it absorbs (when jumping to a higher level) or emits (lower) a photon with an energy  $h\nu$  equal to the difference between the energy levels. This gives rise to a characteristic spectrum which can be observed in stars.

For example, when an electron makes the transition from  $n = 3$  to  $n = 1$ , it can do so directly, or via the pair of transitions  $3 \rightarrow 2 \rightarrow 1$ . In the former case

$$E_{3 \rightarrow 1} = 13.6 \text{ eV} \left( 1 - \frac{1}{3^2} \right) = 12.09 \text{ eV} \quad (2)$$

which corresponds to a wavelength of

$$\lambda_{3 \rightarrow 1} = \frac{c}{\nu} = \frac{hc}{E} = \frac{1240 \text{ eV nm}}{12.09 \text{ eV}} = 102.6 \text{ nm} \quad (3)$$

This is in the ultraviolet.

The pair of transitions gives

$$E_{3 \rightarrow 2} = 13.6 \text{ eV} \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = 1.89 \text{ eV} \quad (4)$$

$$\lambda_{3 \rightarrow 2} = \frac{1240}{1.89} = 656.47 \text{ nm (visible)} \quad (5)$$

$$E_{2 \rightarrow 1} = 13.6 \text{ eV} \left( 1 - \frac{1}{2^2} \right) = 10.2 \text{ eV} \quad (6)$$

$$\lambda_{2 \rightarrow 1} = \frac{1240}{10.2} = 121.6 \text{ nm (ultraviolet)} \quad (7)$$

Transitions to and from  $n = 1$  give the Lyman series (all in the ultraviolet); to and from  $n = 2$  (from higher energy levels) give the Balmer series (visible) and to and from  $n = 3$  (from higher energy levels) give the Paschen series (infrared). For all series, the most energetic photons come from transitions from essentially infinite  $n$  to the base level. For Lyman, for example,

$$E_{n \rightarrow 1} = 13.6 \text{ eV} \left( 1 - \frac{1}{n^2} \right) \quad (8)$$

so the *series limit* occurs as  $n \rightarrow \infty$  and is 13.6 eV. This corresponds to a wavelength of

$$\lambda_1 = \frac{1240}{13.6} = 91.18 \text{ nm (ultraviolet)} \quad (9)$$

For Balmer, the series limit is

$$E_{n \rightarrow 2} = 13.6 \text{ eV} \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \rightarrow 3.4 \text{ eV} \quad (10)$$

$$\lambda_2 = \frac{1240}{3.4} = 364.7 \text{ nm (slightly ultraviolet)} \quad (11)$$

For Paschen,

$$E_{n \rightarrow 3} = 13.6 \text{ eV} \left( \frac{1}{3^2} - \frac{1}{n^2} \right) \rightarrow 1.51 \text{ eV} \quad (12)$$

$$\lambda_3 = \frac{1240}{1.51} = 820.6 \text{ nm (slightly infrared)} \quad (13)$$

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