DE BROGLIE WAVES

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problems 5.12 - 5.13.

Historically, the idea that massive particles such as electrons exhibit wave behaviour was due to Louis de Broglie in his 1924 PhD thesis. De Broglie drew an analogy between the behaviour of light and the behaviour of every other type of matter. From Planck's relation for photons

$$E = h\nu \tag{1}$$

and the relativistic equation relating the energy and momentum of a photon (with c restored in the equation):

$$E = cp \tag{2}$$

we can use the fact that, for photons, $\lambda\nu = c$ to get

$$\lambda = \frac{hc}{E} = \frac{h}{p} \tag{3}$$

De Broglie proposed that this relation applied to *all* particles, whether they be massless or not.

Essentially, de Broglie was anticipating the free particle solution of the Schrödinger equation, which appeared in 1926. The free particle solution is, in one dimension:

$$\psi = Ae^{ikx} + Be^{-ikx} \tag{4}$$

where

$$k = \frac{2\pi}{\lambda} = \frac{\sqrt{2mE}}{\hbar} \tag{5}$$

Since $E = \frac{1}{2}mv^2$ for a (non-relativistic) free particle, $\sqrt{2mE}/\hbar = 2\pi mv/h = 2\pi p/h$ which just restates de Broglie's formula 3.

Things aren't quite this simple, of course, because the stationary state solutions of the free particle equation are not normalizable, so any real particle is actually a superposition of stationary states with a spread of wavelengths. However, for an experiment in which a beam of electrons is travelling through a vacuum in a large volume (such as a cathode ray tube), the stationary state is a good approximation.

Example 1. If an electron in a (old-fashioned, now) cathode ray TV set is travelling at 5×10^{7} m s$^{-1}=0.167c$, its momentum is

$$p=\gamma mv=1.014\left(9.10938291\times10^{-31}\right)\left(5\times10^{7}\right)=4.619\times10^{-23}\text{ kg m s}^{-1} \tag{6}$$

This gives a de Broglie wavelength of

$$\lambda=\frac{h}{p}=1.434\times10^{-11}\text{ m}=0.01434\text{ nm} \tag{7}$$

This is much shorter than visible light (around 500 nm) and, since the resolving power of a microscope is roughly the same as the wavelength of the light used to illuminate the specimen, this shows why an electron microscope is capable of much higher magnification than an optical microscope.

Example 2. In Bohr's semi-classical model of the hydrogen atom, the electron is allowed only discrete values $n\hbar$ of angular momentum. Bohr, however, assumed that the electron is a particle, not a wave. If we instead look at the electron as a wave, then a plausible assumption is that, for a given orbital radius r, the circumference of the orbit $2\pi r$ must be an integral number of wavelengths $n\lambda$ in order for the electron not to destructively interfere with itself. That is

$$\begin{aligned}2\pi r &= n\lambda &(8)\\ &= n\frac{h}{p} &(9)\end{aligned}$$

The angular momentum $\mathbf{L}=\mathbf{r}\times\mathbf{p}$ is therefore (assuming that $\mathbf{r}\perp\mathbf{p}$ which is true of an object travelling in a circular orbit):

$$L=rp=n\frac{h}{2\pi}=n\hbar \tag{10}$$

Thus the wave nature of the electron gives rise to the quantization of angular momentum that Bohr assumed in his model.