UNCERTAINTY PRINCIPLE: VISUALIZATION WITH FOURIER SERIES

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 5, Problem 5.18.

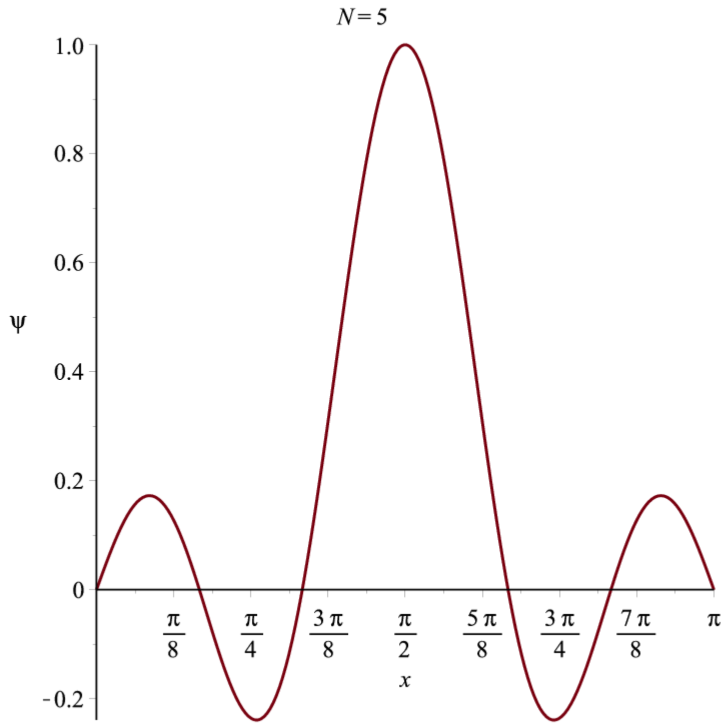
To represent a real free particle, we need to write its wave function as the superposition of plane waves of different wavelengths, in order that the overall wave function is normalizable. In general, we need to use a Fourier transform to do this (that is, we need to integrate over a continuous range of wavelengths). However, we can get a feel for the procedure by using a Fourier series instead, in which we sum over a finite number of discrete wavelengths.

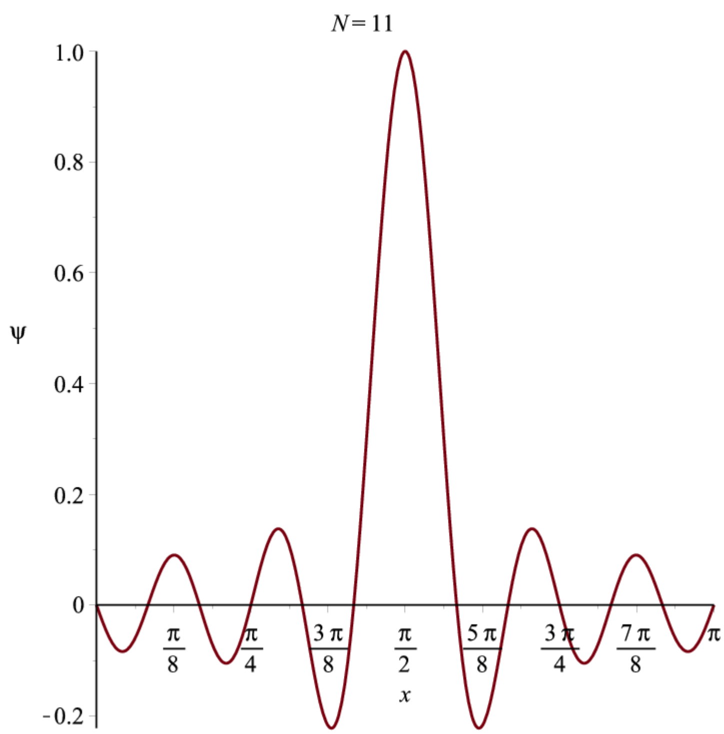
We'll have a look at the series:

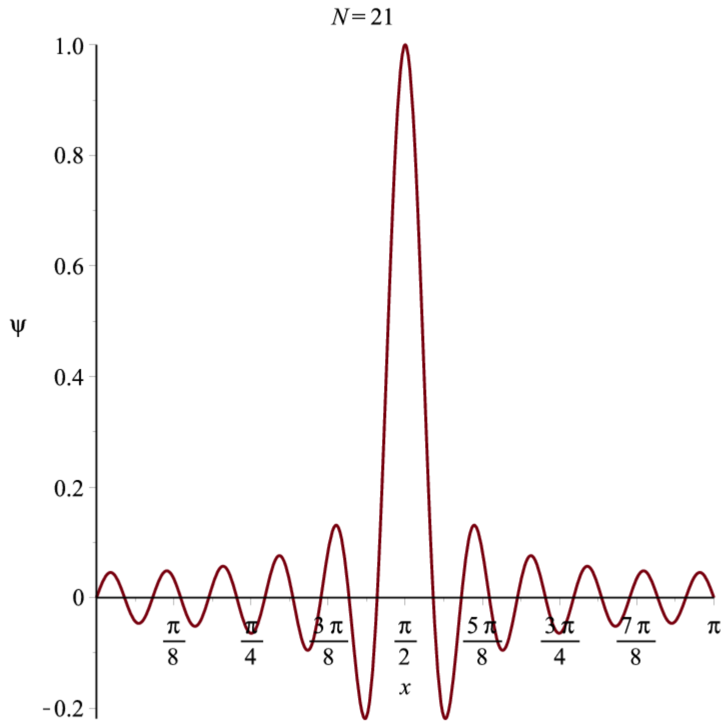
$$\Psi = \frac{2}{N+1}\left[\sin x-\sin 3x+\sin 5x-\ldots\pm\sin Nx\right] \quad (1)$$

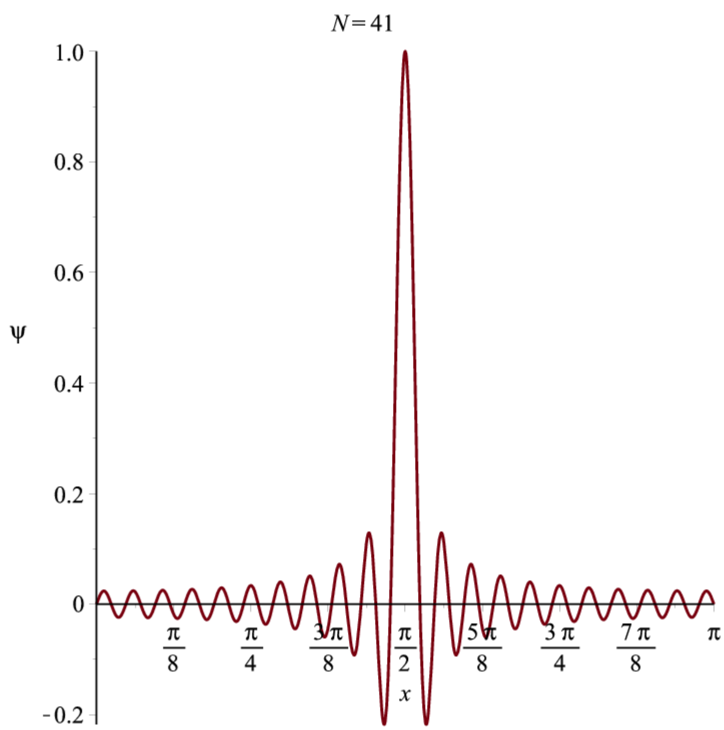
$$= \frac{2}{N+1}\sum_{n=1,odd}^{N}(-1)^{(n-1)/2}\sin nx \quad (2)$$

This defines a wave packet in the interval $x\in[0,\pi]$ which peaks at $x=\pi/2$. Using Maple, we can generate plots of Ψ for various values of N:


N = 5
1.0
0.8
0.6
ψ
0.4
0.2
0
- 0.2
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8
π
x


N = 11
1.0
0.8
0.6
ψ
0.4
0.2
0
- 0.2
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8
π
x


N = 21
1.0
0.8
0.6
ψ
0.4
0.2
0
-0.2
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8
π
x

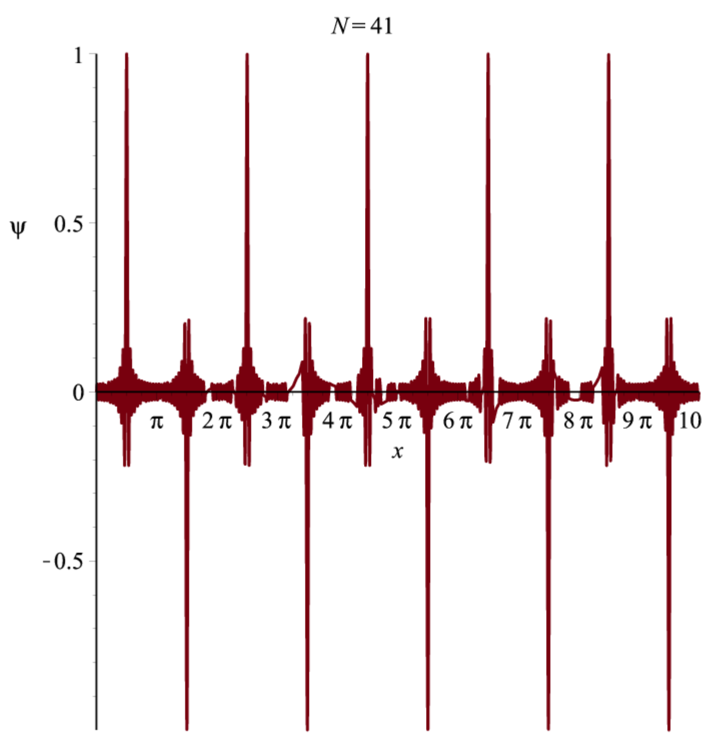

N = 41
1.0
0.8
0.6
ψ
0.4
0.2
0
-0.2
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8
π
x

If we define the width of the central peak as the range Δx between the values of x for which $\Psi = 0.5$, we can use Maple's fsolve command to find these values of x. We get

N	Δx
5	0.638
11	0.317
21	0.172
41	0.090

The location of a particle represented by the wave function Ψ is more accurately known for higher N. Conversely, the momentum is better known for lower N, since there are fewer wavelengths (hence, fewer energies and momenta) contributing to Ψ if N is smaller. This is a graphic representation of the position-momentum uncertainty principle.

By the way, the equation 2 defines a wave packet only for the region $0 \leq x \leq \pi$. If we use this equation for a larger interval, say $0 \leq x \leq 10\pi$, we get a graph like this:



To define a single wave packet over an interval $0 \leq x \leq A\pi$ we need to modify 2 like this:

$$\Psi = \frac{2}{N+1} \sum_{n=1,odd}^{N} (-1)^{(n-1)/2} \sin \frac{nx}{A} \tag{3}$$