LENSMAKER'S EQUATION

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 6, Problems 6.3 - 6.4.

A lens whose two surfaces are spheroidal (sections of a sphere, as opposed to something more exotic like a paraboloid) can have its focal length determined if the radii of curvature of the two surfaces of the lens are known (along with the index of refraction n of the lens material). By convention, a convex surface has a positive radius of curvature and a concave surface a negative radius of curvature. The equation giving the focal length is known as the *lensmaker's formula* (which we'll just quote here, but it's essentially derived from Snell's law of refraction, as you might expect):

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \tag{1}$$

where $R_{1,2}$ are the two radii of curvature.

If we make a compound lens by putting two lenses in contact in such a way that one lens has a side R_1 and a convex side with curvature R_2 and the other lens has a concave side with curvature $-R_2$ and another side with curvature R_3, then we've essentially produced a single lens with sides of R_1 and R_3, so its focal length is

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_3}\right)=\frac{1}{f_1}+\frac{1}{f_2} \tag{2}$$

At this point, Carroll & Ostlie ask us to calculate what would happen if we produced a compound lens like this, but where the two component lenses had different indices of refraction. The formula 2 doesn't work in this case, since the two terms involving R_2 don't cancel. I'm assuming that we're still supposed to use this formula, however, but possibly treating it as an approximation. In any case, we're supposed to show that if the two indices of refraction $n_{1\lambda}$ and $n_{2\lambda}$ depend on wavelength differently, we can build the compound lens such that the focal lengths at two distinct wavelengths are equal. Starting with 1, we have for the compound lens

$$\frac{1}{f} = \frac{1}{f_1}+\frac{1}{f_2} \tag{3}$$
$$= A\left(n_{1\lambda}-1\right)+B\left(n_{2\lambda}-1\right) \tag{4}$$

where

$$A \equiv \left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{R_1+R_2}{R_1R_2} \tag{5}$$
$$B \equiv \left(\frac{1}{R_3}-\frac{1}{R_2}\right)=\frac{R_2-R_3}{R_2R_3} \tag{6}$$

If the focal lengths for the two wavelengths λ and μ are to be equal then

$$A\left(n_{1\lambda}-1\right)+B\left(n_{2\lambda}-1\right)=A\left(n_{1\mu}-1\right)+B\left(n_{2\mu}-1\right) \tag{7}$$
$$\frac{n_{2\mu}-n_{2\lambda}}{n_{1\lambda}-n_{1\mu}}=\frac{A}{B} \tag{8}$$
$$=\frac{R_3\left(R_1+R_2\right)}{R_1\left(R_2-R_3\right)} \tag{9}$$

If the materials in the lenses are known, then the expression involving the indices of refraction is known, so it is up to the lensmaker to arrange the radii of curvature to satisfy that relation.

If the two lenses have the *same* index of refraction, then $n_1=n_2$ and this equation reduces to

$$\frac{R_3\left(R_1+R_2\right)}{R_1\left(R_2-R_3\right)} = -1 \tag{10}$$
$$R_3 = -R_1 \tag{11}$$

That is, one side is convex and the other concave, but the curvatures are the same so the lens is just an ordinary sheet of glass of constant thickness, which would pass all wavelengths the same way.

As the condition 9 was derived so that the focal lengths for two specific wavelengths would be the same, there's no guarantee that it would hold for other wavelengths. To do so, the ratio $\frac{n_{2\mu}-n_{2\lambda}}{n_{1\lambda}-n_{1\mu}}$ would have to be independent of wavelength.