SINGLE SLIT DIFFRACTION

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 6, Problem 6.6.

When monochromatic light passes through a single slit of finite width and is projected on a screen some distance from the slit, a diffraction pattern appears. The analysis of this pattern is similar to that for a diffraction grating, except this time we're dealing with only a single slit, and the slit has a non-zero width.

We'll start by dividing the slit in half. Now suppose we choose a location on the screen such that a ray from the top of the slit travels exactly half a wavelength $\lambda/2$ further to reach the screen than a ray from the middle of the slit. In this case, the two rays interfere destructively, cancelling each other out. Now consider a ray from a distance ϵ below the top of the slit. It will travel a distance $\lambda/2$ farther to reach the screen than a ray starting from $\frac{D}{2}+\epsilon$. We can continue the argument for all values of ϵ from 0 up to $\frac{D}{2}$, so the point on the screen that we've been considering will get no illumination, as all the rays from the slit cancel in pairs.

Suppose we divide up the width D of the slit into an even number $2m$ of segments. Then we can repeat the above argument for a different point on the screen such that a ray from the first segment travels $\lambda/2$ further to reach the screen than a ray in the second segment, and so on, pairing the third and fourth segments, fifth and sixth, etc. Thus for each $m=1,2,3,\ldots$ we get a dark fringe on the screen. [Note that the argument doesn't work for an odd number of segments, since the rays from the last segment can't be paired with another segment to produce destructive interference.]

We can find the points on the screen in terms of the angle θ between the direction of the rays hitting the screen and the normal to the screen. [Although rays from different locations in the slit travel at slightly different angles to reach the same point on the screen, in practice, the slit width D is much less than the distance to the screen, so the rays are almost parallel.] By drawing a diagram (see Fig. 6.7 in Carroll & Ostlie) we see that the path difference is $\frac{D}{2m}\sin\theta$ so the condition for a dark fringe is

$$\begin{aligned}\frac{D}{2m}\sin\theta &= \frac{\lambda}{2} && (1)\\ \sin\theta &= m\frac{\lambda}{D} && (2)\end{aligned}$$

By a process of adding up wave amplitudes from across the slit, we can work out a formula for the intensity of light at angle θ:

$$I(\theta) = I_0\frac{\sin^2\gamma}{\gamma^2} \tag{3}$$

where

$$\gamma \equiv \frac{\pi D}{\lambda}\sin\theta \tag{4}$$

[This formula bears a superficial resemblance to the intensity of light from a diffraction grating with N slits: $I = I_0\frac{\sin^2 N\gamma}{\sin^2\gamma}$. However, the diffraction grating formula ignores the width of each individual slit, so setting $N = 1$ in that formula gives $I = I_0$ with no diffraction pattern at all.]

Because $\lim_{\gamma\to 0}\frac{\sin\gamma}{\gamma} = 1$, $I_0 = I(0)$. This can also be done using l'Hôpital's rule:

$$\lim_{\gamma\to 0}\frac{\sin\gamma}{\gamma} = \lim_{\gamma\to 0}\frac{\cos\gamma}{1} = 1 \tag{5}$$

To get a feel for the spacing of the fringes, suppose the slit has a width of $D = 1.0\ \mu$m and $\lambda = 500$ nm. Then from 2 with $m = 1$ we have

$$\begin{aligned}\sin\theta &= \frac{5\times 10^{-7}}{10^{-6}} = 0.5 && (6)\\ \theta &= 30^\circ && (7)\end{aligned}$$

In this case, the central peak is quite broad. In practice, if we were observing two light sources through such a slit, they would need to be separated by more than 30° to be resolvable on the screen. Increasing the slit width improves the resolution as it decreases the angle θ at which the minimum occurs, and thus it reduces the width of the central peak, giving a sharper image.

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