DIFFRACTION FROM A CIRCULAR APERTURE: RAYLEIGH CRITERION

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 6, Problems 6.7 - 6.8.

Most observing devices in astronomy (from the naked eye up to binoculars and telescopes) use a circular aperture to collect light. Just as with a single slit, a circular aperture also diffracts the light that passes through it. As you might expect, the pattern of dark and light fringes (known as the *Airy disk*) is composed of concentric circles, although the derivation is a bit more complex than in the single slit case. The formula turns out to be

$$I(\theta)=I_0\left(\frac{2J_1(\gamma)}{\gamma}\right)^2 \tag{0.1}$$

where J_1 is the Bessel function of the first kind of order 1 with

$$\gamma\equiv\frac{\pi D}{\lambda}\sin\theta \tag{0.2}$$

and D is the diameter of the aperture. The first minimum occurs at the first zero of $J_1(\gamma)$, which occurs at

$$\gamma_1=3.8317 \tag{0.3}$$

which gives an angle of

$$\sin\theta_1=1.220\frac{\lambda}{D} \tag{0.4}$$

This angular separation is usually taken as the criterion for two light sources (like a double star) to be resolvable, and is known as the *Rayleigh criterion*. For small angles, as is usually the case in astronomy, $\sin\theta_1\approx\theta_1$ and the Rayleigh criterion becomes

$$\theta_1=1.220\frac{\lambda}{D} \tag{0.5}$$

Thus the resolution of a telescope improves with increasing aperture size, and with decreasing wavelength of light.

Example 1. For the human eye, with a pupil diameter of 5 mm, the resolving power for green light (in the middle of the visible spectrum) is

$$\theta_1 = 1.220\frac{550\times10^{-9}}{0.005} \tag{0.6}$$
$$= 1.342\times10^{-4}\text{ rad} \tag{0.7}$$
$$= 27.7'' \tag{0.8}$$

The angular diameter of the moon is around half a degree ($1800''$) while that of Jupiter varies between around $30''$ to $50''$depending on how far away it is from Earth. Thus with perfect eyesight and perfect seeing conditions, it should be just possible to make out Jupiter's disk with the naked eye. In practice, of course, even if your eyesight is very good, atmospheric turbulence will usually mess things up, so you won't be able to make out Jupiter's disk. It might be possible to see the disk if you're on the International Space Station, though I don't know if anyone has tried.

Example 2. A common amateur telescope has a mirror diameter of around 8 inches (20 cm). In this case

$$\theta_1 = 1.220\frac{550\times10^{-9}}{0.20} \tag{0.9}$$
$$= 3.355\times10^{-6}\text{ rad} \tag{0.10}$$
$$= 0.7'' \tag{0.11}$$

If this resolution were to be achieved, we could in theory see a crater on the Moon with a diameter of

$$d = R\theta_1 \tag{0.12}$$
$$= \left(3.844\times10^{8}\text{ m}\right)\left(3.355\times10^{-6}\text{ rad}\right) \tag{0.13}$$
$$= 1290\text{ m} \tag{0.14}$$
$$= 1.29\text{ km} \tag{0.15}$$

where R is the semimajor axis of the Moon's orbit. Again, due to atmospheric turbulence, you wouldn't get close to this limit.

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