IMAGE SIZE AT THE FOCAL PLANE; THE PLATE SCALE

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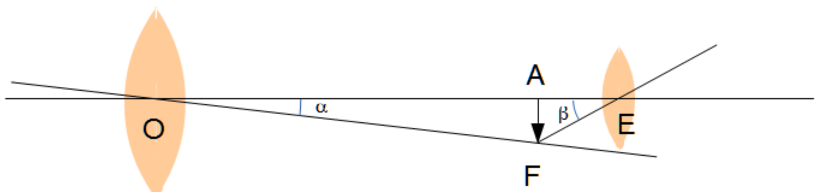
Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 6, Problems 6.9 - 6.10.

The linear size of the image of an object seen through a telescope can be found from a bit of trigonometry.



In the diagram, the image of an upright arrow (to the left of the lens O, not shown in the diagram) is formed at AF. The actual arrow subtends an angle α as seen from the telescope's objective lens O, and will subtend the same angle on the focal plane, which is at a distance f (the focal length) from the lens. The size $y=\overline{\mathrm{AF}}$ is therefore

$$y = f\tan\alpha \tag{0.1}$$

or, for small angles where $\tan\alpha\approx\alpha$

$$y = f\alpha \tag{0.2}$$

For two objects subtending an angle $d\theta$, their linear separation dy at the focal plane is given by

$$\frac{d\theta}{dy} = \frac{1}{f} \tag{0.3}$$

[I'm not sure if this is meant to be an actual differential equation, since for any given telescope, f is a constant anyway so the equation doesn't really say anything more than just $y=f\theta$.] The quantity $\frac{d\theta}{dy}$ is known as the *plate scale*, since it relates the angular separation to the linear separation on a photographic plate placed at the focal plane.

Example 1. It comes in handy even for we amateur astronomers who dabble in taking photographs through a backyard telescope. In that case, the camera replaces the eyepiece. The camera itself doesn't have its own lens; rather it uses an adapter to attach to the telescope so that the telescope serves as a giant telephoto lens (or, more commonly, mirror). For my telescope, $f = 2.8$ m so the plate scale is $\frac{d\theta}{dy} = 0.357$ rad m^{-1}. An image of Jupiter when its angular diameter is $40''$ is therefore

$$dy = f\,d\theta = 0.54 \text{ mm} \tag{0.4}$$

My camera has a sensor size of 23.7 mm by 15.6 mm, and contains 10.20 Megapixels, so the pixel density is 27588 pixels mm^{-2}. An image 0.54 mm in diameter therefore contains 6318 pixels. Thus even with such a small image of around half a millimetre, a digital camera can still get a decent picture.

Example 2. The New Technology Telescope (NTT) in Chile has a 3.58 m primary mirror and a focal ratio (ratio of focal length to diameter) of $f/2.2$. The focal length is therefore

$$f = 3.58 \times 2.2 = 7.876 \text{ m} \tag{0.5}$$

The plate scale is

$$\frac{d\theta}{dy} = 0.127 \text{ rad m}^{-1} \tag{0.6}$$

For a double star such as ε Boo whose components are separated by $2.9''$ the linear separation at the focal plane is

$$dy = f\,d\theta = 7.876 \times \frac{2.9}{3600} \times \frac{\pi}{180} = 0.11 \text{ mm} \tag{0.7}$$

Example 3. The Hubble Space Telescope's Wide Field and Planetary Camera (WF/PC2) consists of four CCD cameras, each of which contains an array of 800×800 pixels. When operating in planetary mode, the WF/PC2 has a focal ratio of $f/28.3$ and a plate scale of $0.0455''$ pixel^{-1}. The telescope's primary mirror has a diameter of 2.4 m. The effective focal length is

$$f = 28.3 \times 2.4 = 67.92 \text{ m} \tag{0.8}$$

The angular size of the field of view of one of the CCD cameras is

$$d\theta = 0.0455 \times 800 = 36.4'' \tag{0.9}$$

Thus on average, Jupiter would just fit into the field of view of one of the cameras. The linear size of one pixel is

$$dy = f\, d\theta = 67.92 \times \frac{0.0455}{3600} \times \frac{\pi}{180} = 0.015 \text{ mm} \tag{0.10}$$

The linear size of one of the cameras is thus around 12 mm.