BINARY STAR ORBITS: RELATION OF SEMIMAJOR AXES

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 7, Problem 7.1.

As a significant fraction of star systems contain double stars, a lot of the effort in studying stellar properties has been concentrated on binary star systems. In a binary system, it's easiest to analyze the orbits of the two stars in the centre of mass frame. In this frame, the two-body problem can be reduced to a one-body problem in which the reduced mass $\mu = m_1 m_2/(m_1+m_2)$ orbits a fixed mass $M = m_1 + m_2$ located at the centre of mass. The reduced mass's distance r from M is an ellipse (for bound orbits) with semimajor axis a and eccentricity e.

The two stars are always on opposite sides of the centre of mass at distances

$$d_1 = \frac{m_2}{M} r \tag{1}$$

$$d_2 = \frac{m_1}{M} r \tag{2}$$

[I'm using d_1 and d_2 instead of r_1 and r_2 to avoid confusion with the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ which give the positions of m_1 and m_2 relative to some arbitrary origin. $d_{1,2}$ are always measured from the centre of mass.]

Since the masses of the stars are constants (in this simple analysis, anyway) the distances $d_{1,2}$ are scaled down copies of r so they too must describe ellipses with the same eccentricity e as the ellipse followed by the reduced mass.

For a point lying on the major axis of an ellipse, the distance from the farthest focus is $2a-p$, where a is the semimajor axis and p is the perihelion distance (distance from the focus to the opposite end of the ellipse). When the two stars are aligned at opposite ends of their respective major axes (that is, when they are farthest apart, or at aphelion), then the distance of star i from the centre of mass is

$$d_{i,a} = 2a_i - p_i \tag{3}$$

where a_i is the semimajor axis of star i and p_i is its perihelion (closest approach) distance. Since $d_1 + d_2 = r$ at every point in the orbit, then at aphelion

$$
\begin{aligned}
r_a &= d_{1,a} + d_{2,a} && (4)\\
&= 2(a_1 + a_2) - (p_1 + p_2) && (5)\\
&= 2a - p && (6)
\end{aligned}
$$

where a is the semimajor axis of the reduced mass and p is its perihelion distance (the last line follows because all the ellipses are similar (in the geometric sense) to each other).

At perihelion

$$
\begin{aligned}
d_{i,p} &= p_i && (7)\\
r_p &= d_{1,p} + d_{2,p} && (8)\\
&= p_1 + p_2 && (9)\\
&= p && (10)
\end{aligned}
$$

Comparing the two results, we see that

$$a = a_1 + a_2 \tag{11}$$

That is, the semimajor axis of μ is the sum of the semimajor axes of the two individual stars. Because the eccentricities are the same, a similar relation holds for the semiminor axes:

$$b = b_1 + b_2 \tag{12}$$

PINGBACKS

Pingback: Binary stars: determining masses and the mass function