

## ECLIPSING BINARY STARS

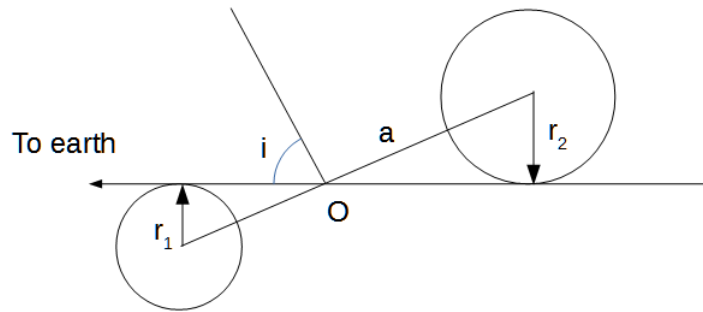
Link to: [physicspages home page](#).

To leave a comment or report an error, please use the [auxiliary blog](#).

Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 7, Problem 7.3.

In attempting to determine the masses of the two stars in a binary system, one of the main problems is that we don't know the angle of inclination  $i$  between the orbital plane of the stars and the plane of the sky. A special case where a constraint can be put on this angle is when  $i$  is close enough to  $90^\circ$  that the stars either partially or totally eclipse each other.

We can find the smallest angle at which the two stars just eclipse each other. The diagram shows the situation:



The stars have radii  $r_1$  and  $r_2$  and their centres are separated by a distance  $a$ , with the plane of the orbit at angle  $i$ . Suppose the distance from  $O$  to the centre of star 1 is  $\frac{a}{2} - \alpha$ , so that the distance from  $O$  to the centre of star 2 is  $\frac{a}{2} + \alpha$ . Then because the two triangles are similar

$$(1) \quad \frac{r_1}{\frac{a}{2} - \alpha} = \frac{r_2}{\frac{a}{2} + \alpha} = \sin\left(\frac{\pi}{2} - i\right) = \cos i$$

Solving the first equation for  $\alpha$  we get

$$(2) \quad \alpha = \frac{a(r_2 - r_1)}{2(r_1 + r_2)}$$

Therefore the minimum inclination angle is given by

$$\begin{aligned}
 (3) \quad \cos i &= \frac{r_2(r_1 + r_2)}{\frac{a}{2}(r_1 + r_2) + \frac{a}{2}(r_2 - r_1)} \\
 (4) \quad &= \frac{r_1 + r_2}{a}
 \end{aligned}$$

[Actually, I suppose this result is a bit more obvious than this derivation shows, since if we move  $r_1$  over so that it's in line with  $r_2$ , we get a right triangle with the side opposite the angle  $\frac{\pi}{2} - i$  of length  $r_1 + r_2$  and hypotenuse  $a$ .]

**Example.** Suppose the two stars have  $r_1 = 10R_S$ ,  $r_2 = R_S$  ( $R_S$  is the radius of the Sun) and  $a = 2$  AU. Then the minimum angle at which an eclipse occurs is

$$\begin{aligned}
 (5) \quad \cos i &= \frac{11R_S}{2 \text{ AU}} \\
 (6) \quad &= \frac{11 \times 696300 \text{ km}}{2 \times 149597871 \text{ km}} \\
 (7) \quad &= 0.0256 \\
 (8) \quad i &= 88.53^\circ
 \end{aligned}$$

If this system is an eclipsing binary, we know  $i$  to within  $1.5^\circ$  so an accurate mass determination should be possible.

#### PINGBACKS

Pingback: Spectroscopic, eclipsing binary stars: mass, radius and temperature

Pingback: YY Sagittarii: an eclipsing binary star

Pingback: Radial velocities of a binary system: computer model

Pingback: YY Sgr: computer model of an eclipsing binary star