# ECLIPSING BINARY STARS

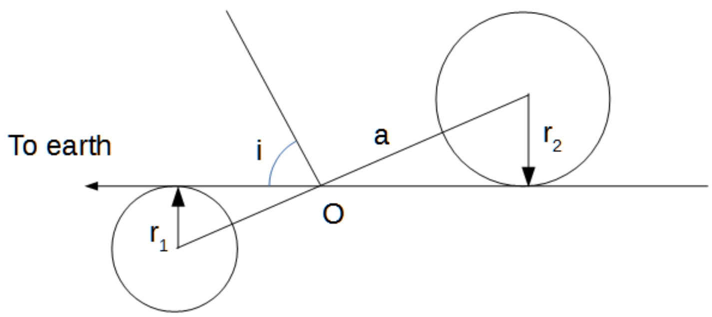
Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 7, Problem 7.3.

In attempting to determine the masses of the two stars in a binary system, one of the main problems is that we don't know the angle of inclination $i$ between the orbital plane of the stars and the plane of the sky. A special case where a constraint can be put on this angle is when $i$ is close enough to $90^{\circ}$that the stars either partially or totally eclipse each other.

We can find the smallest angle at which the two stars just eclipse each other. The diagram shows the situation:




The stars have radii $r_1$ and $r_2$ and their centres are separated by a distance $a$, with the plane of the orbit at angle $i$. Suppose the distance from O to the centre of star 1 is $\frac{a}{2}-\alpha$, so that the distance from O to the centre of star 2 is $\frac{a}{2}+\alpha$. Then because the two triangles are similar

$$\frac{r_1}{\frac{a}{2}-\alpha}=\frac{r_2}{\frac{a}{2}+\alpha}=\sin\left(\frac{\pi}{2}-i\right)=\cos i \tag{1}$$

Solving the first equation for $\alpha$ we get

$$\alpha=\frac{a\left(r_2-r_1\right)}{2\left(r_1+r_2\right)} \tag{2}$$

Therefore the minimum inclination angle is given by

$$\cos i = \frac{r_2\left(r_1+r_2\right)}{\frac{a}{2}\left(r_1+r_2\right)+\frac{a}{2}\left(r_2-r_1\right)} \tag{3}$$

$$= \frac{r_1+r_2}{a} \tag{4}$$

[Actually, I suppose this result is a bit more obvious than this derivation shows, since if we move $r_1$ over so that it's in line with $r_2$, we get a right triangle with the side opposite the angle $\frac{\pi}{2}-i$ of length $r_1+r_2$ and hypotenuse $a$.]

**Example.** Suppose the two stars have $r_1 = 10R_S$, $r_2 = R_S$ ($R_S$ is the radius of the Sun) and $a = 2$ AU. Then the minimum angle at which an eclipse occurs is

$$\cos i = \frac{11R_S}{2\text{ AU}} \tag{5}$$

$$= \frac{11\times 696300\text{ km}}{2\times 149597871\text{ km}} \tag{6}$$

$$= 0.0256 \tag{7}$$

$$i = 88.53^{\circ} \tag{8}$$

If this system is an eclipsing binary, we know $i$ to within $1.5^{\circ}$ so an accurate mass determination should be possible.

## PINGBACKS

Pingback: Spectroscopic, eclipsing binary stars: mass, radius and temperature

Pingback: YY Sagittarii: an eclipsing binary star

Pingback: Radial velocities of a binary system: computer model

Pingback: YY Sgr: computer model of an eclipsing binary star