SIRIUS BINARY STAR SYSTEM

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 7, Problem 7.4.

As an example of using measurements of a binary star system's properties to determine some of its physical characteristics, we'll look at Sirius. Sirius is a visual binary (that is, both stars are visible directly) with a period of $P = 49.94$ years. The parallax of Sirius is $p = 0.37921'' \pm 0.00158''$. If we take the inclination angle to be $i = 0°$, that is, the orbital plane of the Sirius system is in the plane of the sky, then the angular semimajor axis of the reduced mass is $\alpha = 7.61''$. The ratio of the distances of Sirius A (the bright star) and Sirius B (the white dwarf companion) from the centre of mass is $a_A/a_B = 0.466$.

From the parallax, we can get the distance:

$$d = \frac{1}{p} = 2.637 \text{ pc} = 8.597 \text{ ly} \tag{1}$$

We can then find the masses of the two stars. The mass ratio is

$$\frac{m_B}{m_A} = \frac{a_A}{a_B} = 0.466 \tag{2}$$

From Kepler's third law we can get the sum of the masses:

$$m_A + m_B = \frac{4\pi^2}{GP^2} a^3 \tag{3}$$

where a is the semimajor axis of the centre of mass. Since we know the angular semimajor axis and the distance, we have

$$\begin{aligned} a &= \alpha d && (4) \\ &= 7.61'' \frac{\pi}{3600 \times 180} \times 8.597 && (5) \\ &= 3.172 \times 10^{-4} \text{ ly} && (6) \\ &= 3.00 \times 10^{12} \text{ m} && (7) \end{aligned}$$

The period is $P = 1.576 \times 10^9$ s so the sum of the masses is

$$m_A + m_B = \frac{4\pi^2 \left(3.00 \times 10^{12}\right)^3}{\left(6.67 \times 10^{-11}\right)\left(1.576 \times 10^9\right)^2} \tag{8}$$
$$= 6.434 \times 10^{30} \text{ kg} \tag{9}$$
$$= 3.235 M_S \tag{10}$$

where M_S is the mass of the Sun. From this and 2, we get

$$m_B = 0.466 m_A \tag{11}$$
$$m_A = \frac{3.235}{1.466} M_S \tag{12}$$
$$= 2.21 M_S \tag{13}$$
$$m_B = 1.03 M_S \tag{14}$$

[These values are somewhat higher than the currently accepted values of $m_A = 2.02 M_S$ and $m_B = 0.978 M_S$. The discrepancy arises from the currently measured angular semimajor axis being smaller at $\alpha = 7.50 \pm 0.04''$, the period is slightly longer at $P = 50.09 \pm 0.055$ yr and the angle of inclination not being 0, but rather $i = 136.53 \pm 0.43°$.]

We've already worked out the luminosity of Sirius A from its absolute bolometric magnitude of $M_A = +1.36$ and that of the Sun ($M_S = +4.74$):

$$\frac{L_A}{L_S} = 100^{(M_S - M_A)/5} = 22.49 \tag{15}$$

For the companion $M_B = +8.79$:

$$\frac{L_B}{L_S} = 100^{(M_S - M_B)/5} = 0.024 \tag{16}$$

Given the surface temperature of Sirius B as $T_B = 24,790$ K and treating it as a blackbody, we can estimate its radius R_B from the formula

$$L_B = 4\pi R_B^2 \sigma T_B^4 \tag{17}$$

where $\sigma = 5.670373 \times 10^{-8}$ W m^{-2}K^{-4} is the Stefan-Boltzmann constant. The luminosity of the Sun is

$$L_S = 3.839 \times 10^{26} \text{ watts} \tag{18}$$

so

$$
\begin{aligned}
R_B &= \sqrt{\frac{L_B}{4\pi\sigma T_B^4}} && (19)\\
&= \sqrt{\frac{0.024L_S}{4\pi\sigma T_B^4}} && (20)\\
&= \sqrt{\frac{0.024\times 3.839\times 10^{26}}{4\pi\left(5.670373\times 10^{-8}\right)(24790)^4}} && (21)\\
&= 5.85\times 10^6\text{ m} && (22)
\end{aligned}
$$

This is slightly less than the radius of the Earth (6.371×10^6 m) and only 0.0084 times the radius of the Sun (6.963×10^8 m).