SIRIUS BINARY STAR SYSTEM

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 7, Problem 7.4.

As an example of using measurements of a binary star system's properties to determine some of its physical characteristics, we'll look at Sirius. Sirius is a visual binary (that is, both stars are visible directly) with a period of $P=49.94$ years. The parallax of Sirius is $p=0.37921''\pm0.00158''$. If we take the inclination angle to be $i=0^{\circ}$, that is, the orbital plane of the Sirius system is in the plane of the sky, then the angular semimajor axis of the reduced mass is $\alpha=7.61''$. The ratio of the distances of Sirius A (the bright star) and Sirius B (the white dwarf companion) from the centre of mass is $a_A/a_B=0.466$.

From the parallax, we can get the distance:

$$d=\frac{1}{p}=2.637\text{ pc}=8.597\text{ ly} \tag{1}$$

We can then find the masses of the two stars. The mass ratio is

$$\frac{m_B}{m_A}=\frac{a_A}{a_B}=0.466 \tag{2}$$

From Kepler's third law we can get the sum of the masses:

$$m_A+m_B=\frac{4\pi^2}{GP^2}a^3 \tag{3}$$

where a is the semimajor axis of the centre of mass. Since we know the angular semimajor axis and the distance, we have

$$\begin{aligned}
a &= \alpha d && (4)\\
&= 7.61''\frac{\pi}{3600\times180}\times8.597 && (5)\\
&= 3.172\times10^{-4}\text{ ly} && (6)\\
&= 3.00\times10^{12}\text{ m} && (7)
\end{aligned}$$

The period is $P=1.576\times10^{9}$ s so the sum of the masses is

$$
\begin{aligned}
m_A + m_B &= \frac{4\pi^2 \left(3.00 \times 10^{12}\right)^3}{\left(6.67 \times 10^{-11}\right)\left(1.576 \times 10^9\right)^2} && (8) \\
&= 6.434 \times 10^{30} \text{ kg} && (9) \\
&= 3.235 M_S && (10)
\end{aligned}
$$

where M_S is the mass of the Sun. From this and 2, we get

$$
\begin{aligned}
m_B &= 0.466 m_A && (11) \\
m_A &= \frac{3.235}{1.466} M_S && (12) \\
&= 2.21 M_S && (13) \\
m_B &= 1.03 M_S && (14)
\end{aligned}
$$

[These values are somewhat higher than the currently accepted values of $m_A = 2.02 M_S$ and $m_B = 0.978 M_S$. The discrepancy arises from the currently measured angular semimajor axis being smaller at $\alpha = 7.50 \pm 0.04''$, the period is slightly longer at $P = 50.09 \pm 0.055$ yr and the angle of inclination not being 0, but rather $i = 136.53 \pm 0.43^\circ$.]

We've already worked out the luminosity of Sirius A from its absolute bolometric magnitude of $M_A = +1.36$ and that of the Sun ($M_S = +4.74$):

$$
\frac{L_A}{L_S} = 100^{(M_S - M_A)/5} = 22.49 \tag{15}
$$

For the companion $M_B = +8.79$:

$$
\frac{L_B}{L_S} = 100^{(M_S - M_B)/5} = 0.024 \tag{16}
$$

Given the surface temperature of Sirius B as $T_B = 24,790$ K and treating it as a blackbody, we can estimate its radius R_B from the formula

$$
L_B = 4\pi R_B^2 \sigma T_B^4 \tag{17}
$$

where $\sigma = 5.670373 \times 10^{-8}$ W m^{-2}K^{-4} is the Stefan-Boltzmann constant. The luminosity of the Sun is

$$
L_S = 3.839 \times 10^{26} \text{ watts} \tag{18}
$$

so

$$R_B = \sqrt{\frac{L_B}{4\pi\sigma T_B^4}} \tag{19}$$

$$= \sqrt{\frac{0.024 L_S}{4\pi\sigma T_B^4}} \tag{20}$$

$$= \sqrt{\frac{0.024 \times 3.839 \times 10^{26}}{4\pi\left(5.670373 \times 10^{-8}\right)(24790)^4}} \tag{21}$$

$$= 5.85 \times 10^6 \text{ m} \tag{22}$$

This is slightly less than the radius of the Earth (6.371×10^6 m) and only 0.0084 times the radius of the Sun (6.963×10^8 m).