SPECTROSCOPIC BINARY STARS: ZETA PHE

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References: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 7, Problem 7.5.

Andersen, J. (1983) *Spectroscopic observations of eclipsing binaries*, Astron. Astrophys. **118**, 255-261.

As an example of estimating the masses of the components of a spectroscopic binary star, we'll look at ζ Phe in the constellation Phoenix. Its observed period is $P = 1.67$ days and the maximum radial velocities observed from Doppler shifts are (we're assuming the orbits are circular, so the velocities of the stars are constant throughout their orbits):

$$v_{1,r} = 121.4 \text{ km s}^{-1} \tag{1}$$

$$v_{2,r} = 247 \text{ km s}^{-1} \tag{2}$$

The formula for the sum of masses is

$$m_1 + m_2 = \frac{P}{2\pi G}\left(\frac{v_{1,r}+v_{2,r}}{\sin i}\right)^3 \tag{3}$$

As we don't know the inclination angle i, the best we can do is find:

$$(m_1+m_2)\sin^3 i = \frac{P}{2\pi G}\left(v_{1,r}+v_{2,r}\right)^3 \tag{4}$$

$$= \frac{(1.67)(24\times 3600)}{2\pi\left(6.67\times 10^{-11}\right)}\left(1.214\times 10^5 + 2.47\times 10^5\right)^3 \tag{5}$$

$$= 1.72\times 10^{31} \text{ kg} \tag{6}$$

To find the mass ratio, we need the ratio of semimajor axes (or just radii, since we're assuming the orbits are circular), which we can get from velocities and period.

$$\begin{aligned}
r_i &= \frac{v_{i,r}}{2\pi}P && (7)\\
\frac{r_1}{r_2} &= \frac{v_{1,r}}{v_{2,r}} = 0.491 && (8)\\
m_2 &= \frac{r_1}{r_2}m_1 = 0.491m_1 && (9)\\
1.491m_1\sin^3 i &= 1.72\times10^{31}\text{ kg} && (10)\\
m_1\sin^3 i &= 1.153\times10^{31}\text{ kg} && (11)\\
&= 5.8M_S && (12)\\
m_2\sin^3 i &= 5.66\times10^{30}\text{ kg} && (13)\\
&= 2.85M_S && (14)
\end{aligned}$$

Using the average value of $\left\langle\sin^3 i\right\rangle = \frac{3\pi}{16}$ that takes into account the Doppler shift selection effect (the fact that the larger the inclination angle, the more likely it is that a spectroscopic binary will be observed), this gives us mass estimates for the components of ζ Phe:

$$\begin{aligned}
m_1 &= 9.85M_S && (15)\\
m_2 &= 4.84M_S && (16)
\end{aligned}$$

These values are much higher than the actual values of $m_1\sin^3 i = 3.92M_S$ and $m_2\sin^3 i = 2.55M_S$ given in the paper by Andersen. The radial velocity values given by Carroll & Ostlie don't seem to take into account the overall radial velocity of the binary star system relative to Earth, which is $R_v = +15.4$ km s^{-1}. Even after subtracting this out, however, the ratio of the radial velocities of the two components is significantly different from those given in Andersen's paper. If we use the radial velocities in Andersen's Table 5 (he calls them K_A and K_B), which are $v_{1,r} = 131.5$ km s^{-1} and $v_{2,r} = 202.6$ km s^{-1}, we get

$$
\begin{aligned}
(m_1+m_2)\sin^3 i &= \frac{P}{2\pi G}\left(v_{1,r}+v_{2,r}\right)^3 && (17)\\
&= \frac{(1.67)\,(24\times 3600)}{2\pi\,(6.67\times 10^{-11})}\left(1.315\times 10^5+2.026\times 10^5\right)^3 && (18)\\
&= 1.28\times 10^{31}\text{ kg} && (19)\\
&= 6.46 M_S && (20)\\
\frac{r_1}{r_2} &= \frac{v_{1,r}}{v_{2,r}} = 0.649 && (21)\\
m_2 &= 0.649 m_1 && (22)\\
m_1\sin^3 i &= 3.92 M_S && (23)\\
m_2\sin^3 i &= 2.54 M_S && (24)
\end{aligned}
$$

Since ζ Phe is an eclipsing binary, i must be very close to 90° so these values are likely very close to the actual masses.