SPECTROSCOPIC, ECLIPSING BINARY STARS: MASS, RADIUS AND TEMPERATURE

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 7, Problem 7.6.

Here's another example of finding the parameters of the two stars in a spectroscopic, eclipsing binary system. The measured values are

$$P = 6.31 \text{ yr} \tag{1}$$

$$v_{A,r} = 5.4 \text{ km s}^{-1} \tag{2}$$

$$v_{B,r} = 22.4 \text{ km s}^{-1} \tag{3}$$

The ratio of masses is

$$\frac{m_A}{m_B} = \frac{v_{B,r}}{v_{A,r}} = 4.15 \tag{4}$$

The sum of masses is

$$m_1 + m_2 = \frac{P}{2\pi G}\left(\frac{v_{A,r} + v_{B,r}}{\sin i}\right)^3 \tag{5}$$

As the binary is eclipsing, we'll assume that $i \approx 90^\circ$. We get

$$m_A + m_B = \frac{(6.31)(365.25 \times 24 \times 3600)}{2\pi (6.67 \times 10^{-11})}\left(5.4 \times 10^3 + 22.4 \times 10^3\right)^3 \tag{6}$$

$$= 1.02 \times 10^{31} \text{ kg} \tag{7}$$

We can now get the individual masses:

$$m_A = 4.15 m_B \tag{8}$$

$$5.15 m_B = 1.02 \times 10^{31} \text{ kg} \tag{9}$$

$$m_B = 1.98 \times 10^{30} \text{ kg} \approx M_S \tag{10}$$

$$m_A = 8.23 \times 10^{30} \text{ kg} = 4.14 M_S \tag{11}$$

Thus the smaller star has a mass roughly that of the Sun.

The radii of the two stars in an eclipsing system can be found by measuring the times of the various stages of the eclipse. Suppose t_a is the time when the eclipse starts, that is, it's the time when one star just starts to go behind the other one. At this point, the light curve just begins to dip. When the first star is completely behind the other one (we're assuming $i = 90°$ so all eclipses are total) at time t_b, the light curve reaches its minimum. If we know the velocity v of one star relative to the other, this time interval tells us the radius of the eclipsed star:

$$r_B = \frac{v}{2}(t_b - t_a) \tag{12}$$

The radius of the other star can be found from how long it takes the eclipsed star to re-emerge. The eclipsed star first touches the edge of the other star (as seen from Earth) at t_a and begins to emerge at t_c so the radius of the eclipsing star (the one in front) is

$$\begin{align} r_A &= \frac{v}{2}(t_c - t_a) \tag{13} \\ &= \frac{v}{2}(t_c - t_b) + r_B \tag{14} \end{align}$$

For our example, the measured times are $t_b - t_a = 0.58$ day and $t_c - t_b = 0.64$ day. The relative velocity is $v = v_{A,r} + v_{B,r}$ (since the velocities are non-relativistic), so

$$\begin{align} r_B &= \frac{(22.4+5.4)\times 10^3}{2}(0.58\times 24\times 3600) \tag{15} \\ &= 6.97\times 10^8 \text{ m} = R_S \tag{16} \\ r_A &= \frac{0.64+0.58}{0.58} r_B \tag{17} \\ &= 1.47\times 10^9 \text{ m} = 2.1 R_S \tag{18} \end{align}$$

The ratio of the temperatures of the two stars can also be estimated by measuring how much the flux received from the binary star dips in the two minima, if we assume the stars behave as blackbodies. If the smaller star B is hotter, then when it is eclipsed by star A, the light curve drops farther than when star B goes in front of star A, since B is brighter than A, and the same area of stellar surface is eclipsed in both cases. The amount of light received from the binary system when both stars are fully visible is

$$B_0 = k(L_A + L_B) \tag{19}$$

where L_j is the luminosity of star j and k is a constant that depends on the distance and other factors such as the amount of dust between the binary and Earth, and so on.

The brightness B_p of the primary minimum, when star B is fully eclipsed, is just star A on its own:

$$B_p = kL_A \tag{20}$$

The brightness B_s of the secondary minimum is the brightness of star B plus that of star A minus the bit that is obscured by star B:

$$\begin{aligned} B_s &= kL_B + k\frac{\pi r_A^2 - \pi r_B^2}{\pi r_A^2} L_A && (21) \\ &= B_0 - \frac{r_B^2}{r_A^2} B_p && (22) \end{aligned}$$

Comparing the two light minima, we get

$$\frac{B_0 - B_p}{B_0 - B_s} = \frac{L_B}{L_A}\frac{r_A^2}{r_B^2} \tag{23}$$

The luminosity is given in terms of the temperature by the Stefan-Boltzmann law:

$$L = 4\pi r^2 \sigma T^4 \tag{24}$$

where $\sigma = 5.670373 \times 10^{-8}$ W m^{-2}K^{-4} is the Stefan-Boltzmann constant and T is the temperature. We thus have

$$\frac{B_0 - B_p}{B_0 - B_s} = \left(\frac{T_B}{T_A}\right)^4 \tag{25}$$

For our example, the brightnesses are measured in magnitudes, with the maximum brightness at magnitude 5.40, the primary minimum at 9.20 and the secondary minimum at 5.44. We can convert these into the flux of light:

$$\begin{aligned} \frac{B_p}{B_0} &= 100^{(5.40-9.20)/5} = 0.030 && (26) \\ \frac{B_s}{B_0} &= 100^{(5.40-5.44)/5} = 0.964 && (27) \end{aligned}$$

The temperature ratio is

$$\frac{T_B}{T_A} = \left[\frac{1-B_p/B_0}{1-B_s/B_0}\right]^{1/4} \tag{28}$$

$$= 2.28 \tag{29}$$

As we'd expect, the smaller, brighter star B is significantly hotter than star A.

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