EXTRASOLAR PLANETS: SOME DETECTION METHODS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 7, Problems 7.10 - 7.14.

The search for extrasolar planets (planets orbiting stars other than the Sun) is a major area of research in astronomy at the moment, with close to 2000 planets (probably more than 2000 by the time you read this) being discovered so far. Many of the techniques used to discover them are the same as those used in the analysis of binary star systems. Variations in the radial velocity of the parent star can lead to the discovery of an invisible companion. As the star's mass is likely to be much larger than that of the planet, this technique is most effective if the mass of the planet is as large as possible, so that it exerts a larger perturbation in the star's velocity. Shorter periods also aid discovery, since several periods can be observed relatively quickly. For example, if we tried to detect Jupiter from a distant solar system, we'd have to wait 11.86 years for one complete period.

As with binary stars, if we don't know the inclination i of the orbit of a planet, all we can calculate is a lower limit on the planet's (and the star's) mass, since all we can measure is $(m_p \sin i)^3 / (m_p + m_s)^2$, where $m_{p,s}$ are the masses of the planet and star.

A couple of the earliest discoveries of exoplanets are those orbiting the stars 51 Peg and HD 168443. For 51 Peg, the lower limit of the planet's mass is $0.45M_J$ (M_J is Jupiter's mass), with a period of 4.23077 days and a semimajor axis of 0.051 AU. Taking the lower limit as the actual mass, and the orbit as circular, we can get an estimate for the mass of the star 51 Peg from the formula

$$\frac{m_2^3}{(m_1+m_2)^2}\sin^3 i = \frac{P}{2\pi G}v_{1,r}^3 \tag{0.1}$$

We take $m_1 = 0.45M_J = 8.541\times 10^{26}$ kg, $\sin i = 1$, $P = 4.23077$ days and

$$v_{1,r} = \frac{2\pi a}{P} = \frac{2\pi\left(0.051\text{ AU}\left(1.496\times 10^{11}\text{ m AU}^{-1}\right)\right)}{4.23077\text{ days}\left(24\times 3600\text{ s day}^{-1}\right)} = 1.31\times 10^{5}\text{ m s}^{-1} \tag{0.2}$$

The mass of the star is then found from

$$\frac{m_2^3}{(8.541\times10^{26}+m_2)^2} = \left(1.31\times10^5\right)^3 \frac{(4.23077)(24)(3600)}{2\pi(6.67\times10^{-11})} \tag{0.3}$$

$$= 1.96\times10^{30}\text{ kg} \tag{0.4}$$

Taking $m_2 \gg m_1$, this result is thus an approximate value for the mass of 51 Peg (solving the cubic equation exactly in Maple gives the same result, to 2 decimal places, anyway). This is $0.986M_S$ so 51 Peg is effectively the mass of the Sun. (The currently accepted value is $1.11M_S$ so this estimate isn't too bad.)

For planet HD 168443c, the values are $m_1 = 16.96M_J = 3.22\times10^{28}$ kg, $P = 1770$ days $= 1.53\times10^8$ s and $a = 2.87$ AU.

$$v_{1,r} = 1.763\times10^4\text{ m s}^{-1} \tag{0.5}$$

$$\frac{m_2^3}{(3.22\times10^{28}+m_2)^2} = \left(1.763\times10^4\right)^3 \frac{1.53\times10^8}{2\pi(6.67\times10^{-11})} \tag{0.6}$$

$$= 2.00\times10^{30}\text{ kg} \tag{0.7}$$

Solving exactly gives

$$m_2 = 2.06\times10^{30}\text{ kg} \approx 1.04M_S \tag{0.8}$$

In the case of the planet eclipsing the star, we can get an idea of how much the radiant flux received from the star is diminished by considering someone on a distant solar system observing a transit of Jupiter across the Sun. The fraction decrease in flux is just the ratio of the cross sectional areas of Jupiter and the Sun, so (we can neglect the radius of Jupiter's orbit since for an observer sufficiently far away, the Sun and Jupiter appear at essentially the distance):

$$\Delta F = \frac{R_J^2}{R_S^2} = 0.01 \tag{0.9}$$

That is, the Sun's brightness would decrease by only 1%. [Carroll & Ostlie give the temperature of the Sun in the problem, but I can't see that that is relevant to the calculation, since the fractional decrease in flux depends only on how much of the Sun's surface is covered. Doing the same calculation for HD 209458 in their Figure 7.12 gives a reduction in flux of 1.34% which matches the light curve shown in the figure.]

If the planet eclipses its parent star, we can estimate its radius by measuring the times at which the eclipse starts (the edge of the planet just touches the edge of the star) and becomes total (the entire planet is in front of the star). The planet orbiting the star HD 209458 eclipses the star, and the light curves for a couple of transits are shown in Carroll & Ostlie's Figure 7.12. The data are somewhat scattered, but we can estimate that the eclipse starts about 0.05 days before the midpoint and becomes total around 0.03 days before the midpoint. From the given period of $P = 3.525$ days and semimajor axis of $a = 0.0467$ AU, and assuming a circular orbit, its velocity is

$$v = \frac{2\pi a}{P} = 1.441 \times 10^5 \text{ m s}^{-1} \tag{0.10}$$

As it takes about 0.01 days for the radius to move across the edge of the star, the radius of the planet is

$$R = 1.441 \times 10^5 (0.01)(24)(3600) = 1.25 \times 10^8 \text{ m} \approx 1.8 R_J \tag{0.11}$$

This compares with the value of $1.27 R_J$ given in the text, but it's difficult to estimate the eclipse times from such a scattered plot, so I suppose this isn't bad.

PINGBACKS

Pingback: Extrasolar planet detection with light curves: OGLE-TR-56b