IONIZATION IN A HELIUM STAR

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 8, Problem 8.9.

We can apply the Saha equation to a star composed entirely of helium. The equation gives the ratio of the number of atoms in ionization stage $i+1$ to those in stage i:

$$\frac{N_{i+1}}{N_i}=\frac{2kTZ_{i+1}}{P_eZ_i}\left(\frac{2\pi m_ekT}{h^2}\right)^{3/2}e^{-\chi_i/kT} \tag{1}$$

where P_e is the pressure of the free electron gas (that is, electrons that are not attached to any atom) and m_e is the electron mass. The quantities Z_i and Z_{i+1} are the partition functions of the two ionization stages, and χ_i is the energy required to ionize an atom in the ground state of stage i to the ground state of stage $i+1$.

As a helium atom has 2 electrons, it has 3 ionization states: He I (neutral atom), He II (one electron missing) and He III (a bare helium nucleus). We can use the Saha equation to find ratios of numbers of ions in the various states at different temperatures.

The constants in SI units are

$$k = 1.38\times10^{-23}\text{ J K}^{-1} \tag{2}$$
$$m_e = 9.11\times10^{-31}\text{ kg} \tag{3}$$
$$h = 6.63\times10^{-34}\text{ J s} \tag{4}$$
$$1\text{ eV} = 1.6\times10^{-19}\text{ J} \tag{5}$$

The other values are given in problem 8.10 in Carroll & Ostlie:

$$Z_I = 1 \tag{6}$$
$$Z_{II} = 2 \tag{7}$$
$$Z_{III} = 1 \tag{8}$$
$$\chi_I = 24.6\text{ eV} \tag{9}$$
$$\chi_{II} = 54.4\text{ eV} \tag{10}$$

Plugging in the numbers gives

T (K)	N_{II}/N_I	N_{III}/N_{II}
5000	1.85×10^{-18}	4.16×10^{-49}
15000	0.991	2.39×10^{-11}
25000	7213	0.0018

Thus at 15,000 K, $N_I \approx N_{II}$, while we can see that it requires temperatures higher than 25,000 K to get any significant ionization to the He III stage.

We can get expressions for the fraction of helium atoms in each ionization stage as follows.

$$\frac{N_I}{N_t} = \frac{N_I}{N_I+N_{II}+N_{III}} \tag{11}$$

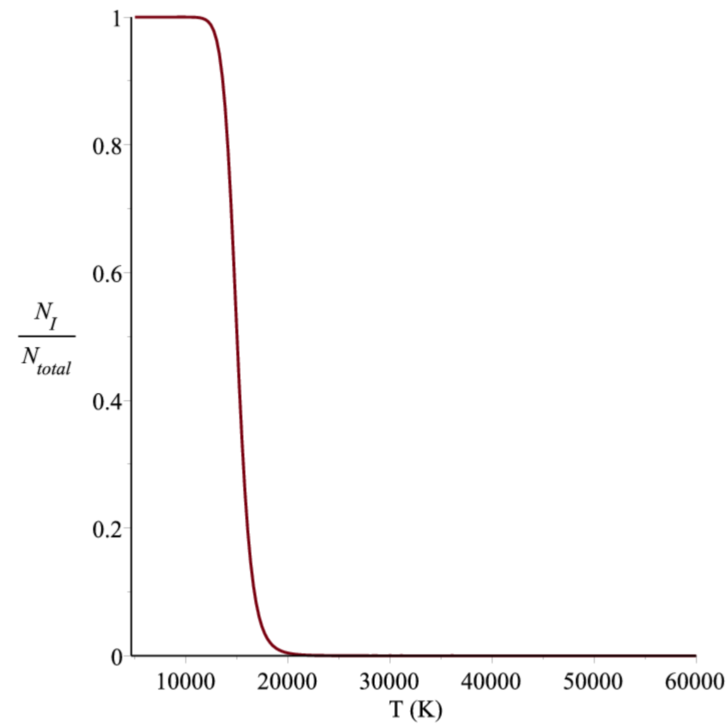
$$= \frac{1}{1+N_{II}/N_I+N_{III}/N_I} \tag{12}$$

$$= \frac{1}{1+N_{II}/N_I+(N_{III}/N_{II})(N_{II}/N_I)} \tag{13}$$

$$\frac{N_{II}}{N_t} = \frac{N_{II}/N_I}{1+N_{II}/N_I+(N_{III}/N_{II})(N_{II}/N_I)} \tag{14}$$

$$\frac{N_{III}}{N_t} = \frac{(N_{III}/N_{II})(N_{II}/N_I)}{1+N_{II}/N_I+(N_{III}/N_{II})(N_{II}/N_I)} \tag{15}$$

In the outer stellar atmosphere, a reasonable value for the pressure is $P_e = 20\ \mathrm{N\ m^{-2}}$. The fractions of each ion are shown in the plots


1
0.8
0.6
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$\frac{N_I}{N_{total}}$
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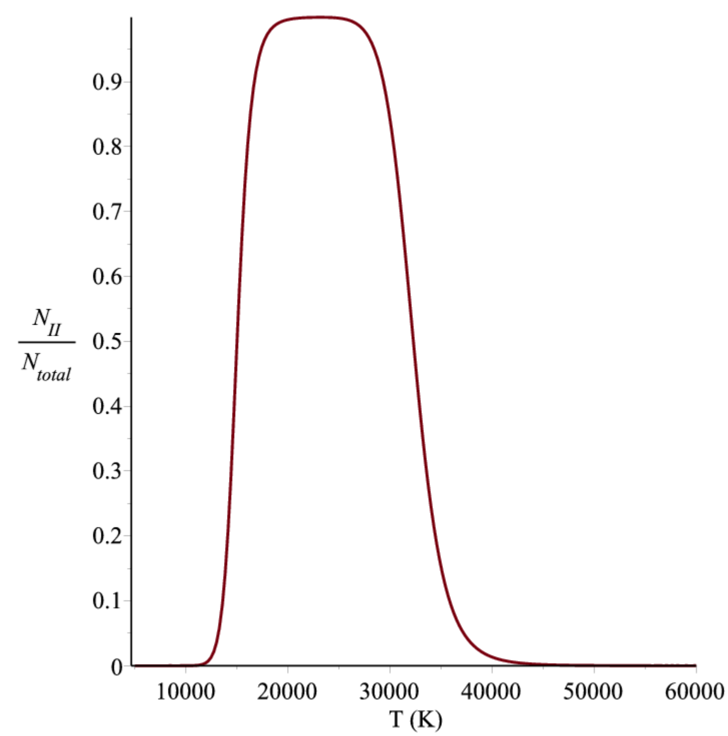
T (K)


0.9
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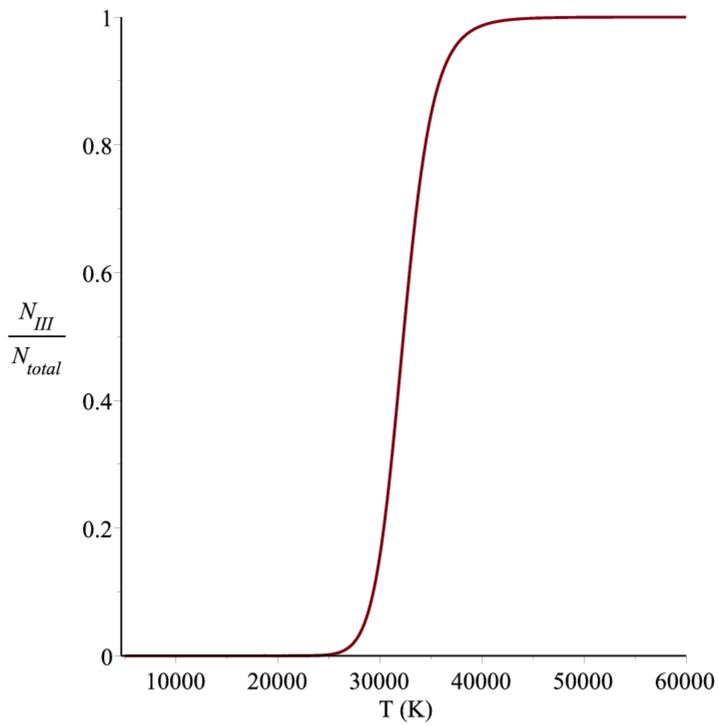
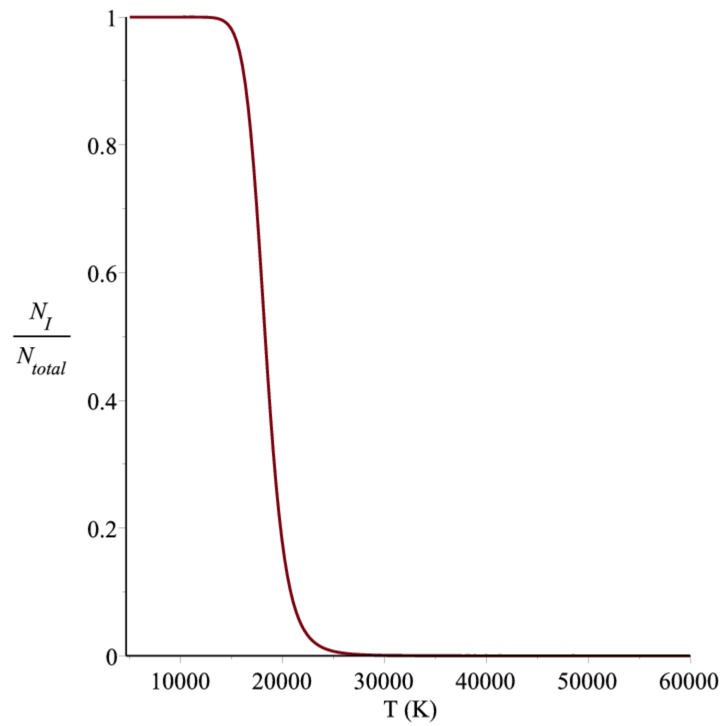
T (K)



As we might expect, the neutral atom dominates at low temperatures, then the He II stage dominates are we raise the temperature. Ultimately, complete ionization occurs as we raise the temperature still further, so that all the helium becomes bare atoms and free electrons.

The temperature at which half the mixture consists of He II (in the stage where neutral helium is being ionized) is $T = 15006$ K. Half the mixture consists of He III at a temperature of $T = 32286$ K.

Deeper inside the star, the pressure is considerably higher, with a value around $P_e = 1000\ \mathrm{N\ m^{-2}}$. We can produce plots for this pressure and we get


1
0.8
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$\frac{N_I}{N_{total}}$
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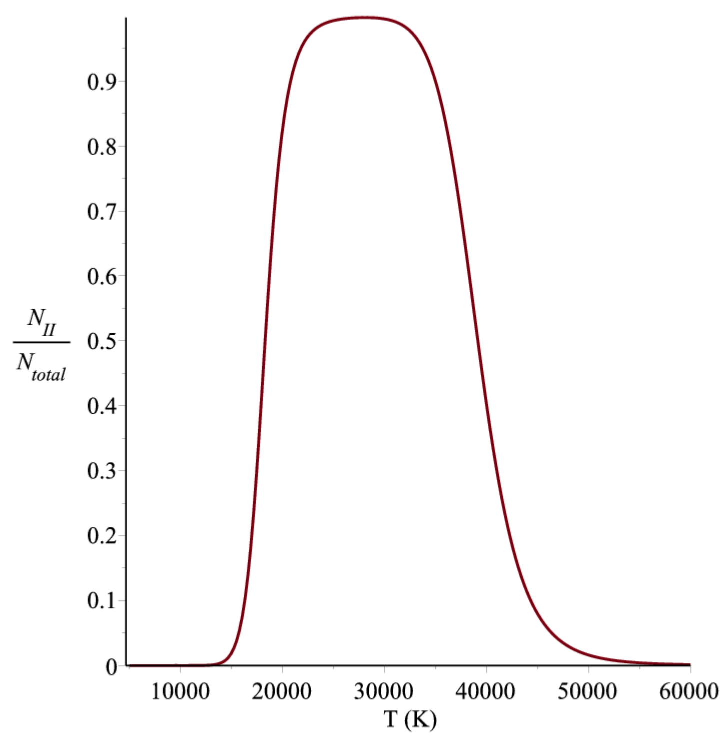
T (K)


0.9
0.8
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$\frac{N_{II}}{N_{total}}$
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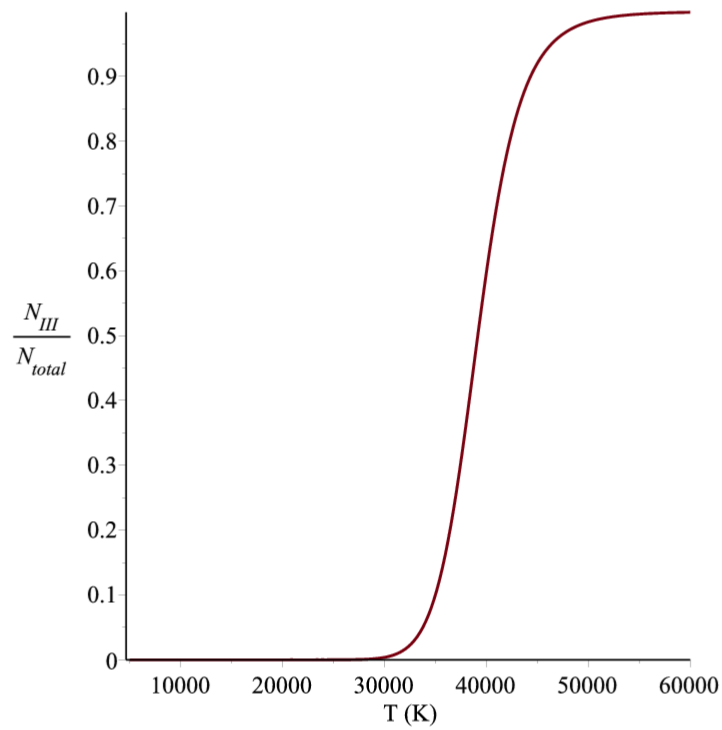
T (K)



The plots are similar to the low pressure plots, with the curves shifted slightly to the higher temperature end. The temperature at which half the mixture consists of He II (in the stage where neutral helium is being ionized) is $T = 18290$ K. Half the mixture consists of He III at a temperature of $T = 39152$ K.