SAHA EQUATION BREAKS DOWN AT THE CENTRE OF THE SUN

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Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 8, Problem 8.12.

Using the Saha equation to calculate ionization ratios for the interior of stars doesn't work very well, as the Saha equation assumes thermodynamic equilibrium and a relatively low density (under 1 kg m^{-3}) for the gas. As an example, we can try to apply the equation to calculate the ionization ratio for the centre of the sun, where the number density of free electrons is

$$n_e = 6.1 \times 10^{31} \text{ m}^{-3} \tag{1}$$

and the temperature is

$$T = 15.7 \times 10^6 \text{ K} \tag{2}$$

The Saha equation gives the ratio of the number of atoms in ionization stage $i+1$ to those in stage i:

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT} \tag{3}$$

where n_e is the number density (per unit volume) of free electrons (that is, electrons that are not attached to any atom) and m_e is the electron mass. The quantities Z_i and Z_{i+1} are the partition functions of the two ionization stages, and χ_i is the energy required to ionize an atom in the ground state of stage i to the ground state of stage $i+1$.

The constants have the values

$$k = 1.38 \times 10^{-23} \text{ J K}^{-1} \tag{4}$$
$$m_e = 9.11 \times 10^{-31} \text{ kg} \tag{5}$$
$$h = 6.63 \times 10^{-34} \text{ J s} \tag{6}$$
$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \tag{7}$$

With $Z_I = 2$ and $Z_{II} = 1$, we get for the fraction of ionized hydrogen atoms with the above values:

$$\frac{N_{II}}{N_t} = \frac{N_{II}/N_I}{1+N_{II}/N_I} \tag{8}$$
$$= 0.71 \tag{9}$$

At the centre of the Sun, it is believed that close to 100% of the hydrogen is ionized, so this value is considerably lower than the true value. Some reasons for this are mentioned in Carroll & Ostlie just before their example 8.1.5, particularly that for high density, the proximity of neighbouring ions can lower the ionization energy.

It's also worth noting that with the density 1, the average distance between electrons is around

$$d \approx \frac{1}{n_e^{1/3}} = 2.5 \times 10^{-11} \text{ m} \tag{10}$$

This is about half the Bohr radius, which is the mean electron-proton distance in neutral hydrogen in the ground state. Thus even 'ionized' atoms could be seen as neutral atoms due to the proximity of electrons and protons.