DIELECTRIC CONSTANT

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Experimentally, the dipole moment of an atom is proportional to the applied electric field (for small fields). Since the polarization of a dielectric is due to individual atoms within the dielectric being given dipole moments, it should come as no surprise that the polarization density **P** of a dielectric is also proportional to the applied field. The relation is written as

$$\mathbf{P}=\epsilon_0\chi_e\mathbf{E} \tag{1}$$

where χ_e is the *electric susceptibility*. Due to the presence of the ϵ_0, χ_e is dimensionless. Not all substances obey such a simple law, but for those that do, they are called *linear dielectrics.*

If this condition holds, we get a simple relationship between the displacement, the field and the polarization. We have

$$\mathbf{D} = \epsilon_0\mathbf{E}+\mathbf{P} \tag{2}$$

$$= \epsilon_0\left(1+\chi_e\right)\mathbf{E} \tag{3}$$

The quantity $1+\chi_e$ is called the *dielectric constant* for a material. The entire proportionality constant is called the *permittivity* ϵ of the material:

$$\epsilon=\epsilon_0\left(1+\chi_e\right) \tag{4}$$

Since a vacuum cannot be polarized at all, $\chi_e=0$ for a vacuum, so that $\epsilon=\epsilon_0$ in that case. This is why ϵ_0 is called the *permittivity of free space*.

As a simple example of how these relations can be used, suppose we have a parallel plate capacitor whose plates are separated by a distance $2a$. On one plate there is a surface charge density of σ and on the other there is $-\sigma$. Between the plates are two slabs of dielectric, each of thickness a. Slab 1 (next to the positive plate) has a dielectric constant of 2 and slab 2 has a dielectric constant of 1.5.

We can begin by finding the displacement **D**. Using Gauss's law for displacement, we can build a little cylindrical Gaussian pillbox of radius r with one end in the positive plate and the other in slab 1. By symmetry **D** is parallel to the axis of the cylinder so there are no contributions to $\mathbf{D}\cdot d\mathbf{a}$

from the sides of the cylinder. (As usual in problems of this type, we're ignoring edge effects on the capacitor.) The end of the cylinder inside the plate will have $\mathbf{D}=0$ (since we're inside a conductor), while the other end will contribute $\pi r^2 D$. The free charge enclosed by the cylinder is $\pi r^2\sigma$ (dielectrics are electrically neutral, so there is no free charge within the dielectric, so that $\nabla\cdot\boldsymbol{D}=\rho_f=0$) so we get

$$\begin{aligned}\int\mathbf{D}\cdot d\mathbf{a} &= Q_f &(5)\\ \pi r^2 D &= \pi r^2\sigma &(6)\\ D &= \sigma &(7)\end{aligned}$$

This result is independent of the dielectric, so it holds everywhere between the plates. If we assume the positive plate lies above the negative one, we have $\mathbf{D}=-\sigma\hat{\mathbf{z}}$ since the displacement vector points from positive to negative.

From the relation above, we can find $\mathbf{E}$. For the two slabs, we have

$$\begin{aligned}\mathbf{E}_1 &= \frac{1}{\epsilon_0\left(1+\chi_e\right)_1}\mathbf{D} &(8)\\ &= -\frac{\sigma}{2\epsilon_0}\hat{\mathbf{z}} &(9)\\ \mathbf{E}_2 &= -\frac{\sigma}{1.5\epsilon_0}\hat{\mathbf{z}} &(10)\end{aligned}$$

The polarization within the two slabs can now be found from 1.

$$\begin{aligned}\mathbf{P}_1 &= \epsilon_0(2-1)\mathbf{E}_1 &(11)\\ &= -\frac{\sigma}{2}\hat{\mathbf{z}} &(12)\\ \mathbf{P}_2 &= -\frac{\sigma}{3}\hat{\mathbf{z}} &(13)\end{aligned}$$

The potential difference between the plates is

$$\begin{aligned}V &= E_1 a+E_2 a &(14)\\ &= \frac{7}{6}\frac{a\sigma}{\epsilon_0} &(15)\end{aligned}$$

The bound charges resulting from the polarization can be found. Since the polarization is uniform within each slab, there is no volume bound charge since $\nabla\cdot\mathbf{P}=0$ everywhere. The surface bound charge is found from

$$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}} \tag{16}$$

Applying this at the boundary between the positive plate and slab 1, we get $\sigma_b = -\sigma/2$. At the boundary between slabs 1 and 2, there is a bound charge of $+\sigma/2$ on slab 1 and $-\sigma/3$ on slab 2 (for a net bound charge of $+\sigma/6$ at the boundary). At the boundary between slab 2 and the negative plate, we have $+\sigma/3$. The dielectrics are electrically neutral, since the surface bound charge on the top surface of each dielectric is balanced by an equal and opposite surface bound chage on the bottom surface.

We can use these bound charges to check the values for the field found above. Choosing a Gaussian cylinder with one end in the positive plate and the other in slab 1, the enclosed charge is $\pi r^2 (\sigma - \sigma/2)$ which is equal to ϵ_0 times the surface integral of the field over the cylinder's end cap which is $\pi r^2 E_1$. Thus $E_1 = \sigma/2\epsilon_0$ (pointing downwards) as before. Doing the same calculation for slab 2 we get $E_2 = 2\sigma/3\epsilon_0$ as before.

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