ELECTRIC DIPOLE

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A common configuration in electrostatics is the *electric dipole*, which consists of two equal and opposite charges $\pm q$ separated by a small distance d. Fig. 1 shows one such setup.

We would like to find the electric field $\boldsymbol{E}$ at some point P at a location $\boldsymbol{r}$ relative to the centre of the dipole. We assume that $r \gg d$ so the observation point is at a distance large relative to the separation of the two charges.

We begin by working out the electric potential at P. The potential due to a single point charge $+q$ is

$$V\left(\boldsymbol{r}'\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r'} \tag{1}$$

where $\boldsymbol{r}'$ is the vector from the charge $+q$ to P, as in Fig. 1.

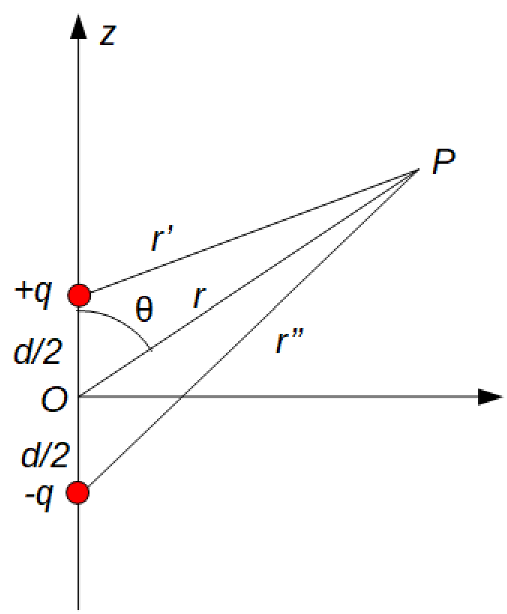

FIGURE 1. Electric dipole.

Given that the angle θ of the vector $\boldsymbol{r}$ relative to the z axis is as shown, we can work out r' by using the cosine rule for a triangle. We have

$$r' = \sqrt{\left(\frac{d}{2}\right)^2 + r^2 - 2\frac{d}{2}r\cos\theta} \tag{2}$$

$$= \sqrt{\frac{d^2}{4} + r^2 - rd\cos\theta} \tag{3}$$

If $r \gg d$, we can neglect the term $d^2/4$ and assume that $rd\cos\theta \ll r^2$, so we can approximate r' by

$$r' \approx \sqrt{r^2 - rd\cos\theta} \tag{4}$$

$$= r\sqrt{1 - \frac{d}{r}\cos\theta} \tag{5}$$

$$\approx r\left(1 - \frac{d}{2r}\cos\theta\right) \tag{6}$$

$$= r - \frac{d}{2}\cos\theta \tag{7}$$

We can then further approximate $1/r'$ as

$$\frac{1}{r'} \approx \frac{1}{r - \frac{d}{2}\cos\theta} \tag{8}$$

$$= \frac{1}{r}\frac{1}{\left(1 - \frac{d}{2r}\cos\theta\right)} \tag{9}$$

$$\approx \frac{1}{r}\left(1 + \frac{d}{2r}\cos\theta\right) \tag{10}$$

$$= \frac{1}{r^2}\left(r + \frac{d}{2}\cos\theta\right) \tag{11}$$

The potential 1 due to $+q$ is therefore approximately

$$V_+ \approx \frac{q}{4\pi\epsilon_0 r^2}\left(r + \frac{d}{2}\cos\theta\right) \tag{12}$$

We can do a similar calculation to find the potential due to charge $-q$, located a distance r'' from P. In this case, the cosine law gives us (using $\cos(\pi - \theta) = -\cos\theta$):

$$r'' = \sqrt{\left(\frac{d}{2}\right)^2 + r^2 - 2\frac{d}{2}r\cos(\pi-\theta)} \tag{13}$$

$$= \sqrt{\left(\frac{d}{2}\right)^2 + r^2 + rd\cos\theta} \tag{14}$$

Thus we get the result that the potential V_- due to $-q$ is just V_+ with $\cos\theta$ replaced by $-\cos\theta$ and $+q$ replaced by $-q$, so we have

$$V_- \approx \frac{-q}{4\pi\epsilon_0 r^2}\left(r - \frac{d}{2}\cos\theta\right) \tag{15}$$

Since the potential due to both charges together is just the sum of the individual potentials, we have

$$V(\boldsymbol{r}) = V_+ + V_- \tag{16}$$

$$= \frac{q}{4\pi\epsilon_0 r^2} d\cos\theta \tag{17}$$

The *electric dipole moment* is a vector defined as the product of the position vector from the negative to the positive charge and the charge on *the positive charge* making up the dipole. For Fig. 1, the dipole moment is

$$\boldsymbol{p} = qd\hat{\boldsymbol{z}} \tag{18}$$

Since

$$\hat{\boldsymbol{z}}\cdot\boldsymbol{r} = r\cos\theta \tag{19}$$

we have

$$V(\boldsymbol{r}) = \frac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\epsilon_0 r^3} \tag{20}$$

$$= \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \tag{21}$$

In spherical coordinates, we can then work out the electric field as the negative gradient of the potential ($\boldsymbol{E} = -\nabla V$), so we have

$$E_r = -\frac{\partial V}{\partial r} = \frac{p\cos\theta}{2\pi\epsilon_0 r^3} \tag{22}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{p\sin\theta}{2\pi\epsilon_0 r^3} \tag{23}$$

$$E_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} = 0 \tag{24}$$

Thus the magnitude of the field is

$$E = \sqrt{E_r^2 + E_\theta^2 + E_\phi^2} = \frac{p}{2\pi\epsilon_0 r^3} = \frac{qd}{2\pi\epsilon_0 r^3} \tag{25}$$

The electric field of a dipole falls off according to an inverse-*cube* law, rather than the inverse-square law that holds for an isolated charge. We'd expect the field to be weaker, since we have contributions from a positive and negative charge combination, so we'd expect the two charges to cancel each other out to some extent.

Notice that the electric field is not entirely radial (unless we are on the z axis), as there is a non-zero E_θ component.

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