ELECTROMAGNETIC WAVES IN CONDUCTORS - ENERGY DENSITY AND INTENSITY

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We can write the electromagnetic wave inside a conductor as (if we orient the axes so that $\mathbf{E}$ is polarized in the x direction)

$$\tilde{\mathbf{E}}(z,t) = \tilde{E}_0 e^{-\kappa z} e^{i(kz-\omega t)}\hat{\mathbf{x}} \tag{1}$$

$$= E_0 e^{-\kappa z} e^{i(kz-\omega t+\delta_E)}\hat{\mathbf{x}} \tag{2}$$

$$\tilde{\mathbf{B}}(z,t) = \frac{\tilde{k}}{\omega}\tilde{E}_0 e^{-\kappa z} e^{i(kz-\omega t)}\hat{\mathbf{y}} \tag{3}$$

$$= \sqrt{\epsilon\mu\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}E_0 e^{-\kappa z} e^{i(kz-\omega t+\delta_E+\phi)}\hat{\mathbf{y}} \tag{4}$$

where

$$\tilde{k} = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}+1}+i\frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1} \equiv k+i\kappa \equiv Ke^{i\phi} \tag{5}$$

The actual fields are the real parts of these equations, so

$$\mathbf{E}(z,t) = E_0 e^{-\kappa z}\cos(kz-\omega t+\delta_E)\hat{\mathbf{x}} \tag{6}$$

$$\mathbf{B}(z,t) = \sqrt{\epsilon\mu\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}}E_0 e^{-\kappa z}\cos(kz-\omega t+\delta_E+\phi)\hat{\mathbf{y}} \tag{7}$$

The energy density in the wave is

$$u = \frac{1}{2}\left(\epsilon E^2+\frac{1}{\mu}B^2\right) \tag{8}$$

Taking the time average (over one cycle) of this we have (since the average of $\cos^2\omega t$ over one cycle $\tau = 2\pi/\omega$ is $\frac{1}{2}$):

$$u=\frac{E_0^2e^{-2\kappa z}}{4}\left(\epsilon+\epsilon\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}\right) \tag{9}$$

For a good conductor, $\sigma \gg \epsilon\omega$ so

$$u \approx \frac{E_0^2e^{-2\kappa z}}{4}\left(\epsilon+\frac{\sigma}{\omega}\right) \tag{10}$$

$$\approx \frac{E_0^2e^{-2\kappa z}}{4}\frac{\sigma}{\omega} \tag{11}$$

From 10, we see that the magnetic contribution (σ/ω) is much larger than the electric contribution (ϵ) for a good conductor.

We can express this in terms of the wave vector k by using 5 for a good conductor.

$$k \approx \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\epsilon\omega}} \tag{12}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} \tag{13}$$

$$\sigma = \frac{2k^2}{\omega\mu} \tag{14}$$

$$u \approx \frac{E_0^2e^{-2\kappa z}}{2}\frac{k^2}{\mu\omega^2} \tag{15}$$

The intensity is the energy crossing a unit area in unit time, which is the energy density times the volume crossing a unit area per unit time, which is

$$I=uv \tag{16}$$

where v is the speed of the wave, which is ω/k so

$$I=\frac{E_0^2e^{-2\kappa z}}{2}\frac{k}{\mu\omega} \tag{17}$$